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The Association
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The Design Of Towers, Masts And Pylons.

By D. BAMBER, A.M.I.C.E., A.M.I.Mech.E.

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INTRODUCTION.

Tower and mast design and construction represents a highly specialized branch of steelwork, and unlike many other branches, little data has been available for design work.

Standard methods of design as used for other lattice structures tend to produce complex systems of calculation and tabulation when applied to towers and more useful methods, as described in succeeding chapters, have been developed. Such systems allow of easy checking of the design stage by stage, and allow the stresses due to special loading conditions to be added later, or super-imposed upon the main design.

Moreover, the precise loading on these structures has been difficult to assess and only recently have full scale tests been carried out on the smaller production type of tower, such as is used for power transmission, to determine the reaction to various loading systems. In fact, in most transmission line schemes, where a large number of suspension, anchor, and corner towers are required a full-scale test is usually specified for each type, the tower having to withstand prescribed loads corresponding to high wind and broken wire conditions multiplied by a safety factor, without premature failure.

Towers may generally be divided into two general types, free standing, and stayed or guyed masts. The latter are little favoured in some countries, as self-supporting towers may be built up to six or eight hundred feet high, economically, take up little ground space and require only small maintenance. The stayed mast, however, is cheaper in first cost and quickly produced and erected by comparison with the free standing type.

The purpose for which the structure is to be used may have a considerable effect upon the general design and also upon the safety factors used. Very tall masts are used mainly for radio transmission, or for carrying power cables over an exceptionally long distance such as a river crossing. In the first case the attachments to the structure may be completely absent, particularly in the case of a stayed mast, or an aerial cable may be attached at or near the top. In the latter case it is evident that a line breakage will generally reduce the load on the structure, for instance, in the case of a single aerial or several attached on one side.

In the case of transmission towers, that is, those used for carrying electric power cables, the conditions are more complex, there being considerable vertical loading due to the weight of the cables, and where long spans have to be considered, a considerable unbalanced longitudinal load due to difference in length of adjacent spans. In these cases the breakage of any line will increase the load on the main structure as torsion loads now come into play, and the design of these types necessitates allowing for a considerable overload.

Further considerations arise in the case of masts subject to dynamic loading, such as the mast of a ship, which is subject to frequent stress reversal. Such cases require a higher safety factor, and owing to the difficulty in assessing the precise loading, established practice provides certain rules which have proved satisfactory when using the traditional materials. In this case, for instance, with steel masts, a factor of five on the crippling load is often used, that is, the mast is designed to buckle at five times the vertical load imposed by the stays and structure weight. Factors for land based masts are differently assessed, and are dealt with under the heading of "Guyed Masts."

Other dynamic loading of unusual type such as earthquake shocks, or the formation of rythmic wind eddies around circular masts are the subject of much controversy. The stresses imposed are dependant upon the vibrational characteristics of the mast as regards mode and frequency, and the results on model tests or site observations are the only reliable guides in this respect.

Steel is the most commonly used structural material, angle or tubular sections being adhered to throughout. If larger, or stronger sections are required, the large diameter steel tube may be used, or lattice girders may be made out of angle sections. Lattice box girders (square cross section) or plane lattice members are commonly used, the latter very often in tower faces where they are set with the strong axis normal to the face, and the weak axis in the plane of the face where adequate and economic bracing can be provided on to the main legs.

Corrosion resistance is very important, and hot dip galvanising is invariably used in Britain, whilst on the Continent painting is usually the only protection used. Much depends on the purpose for which the structure is used, and whether maintenance is easy or not. Transmission line towers must obviously be fully protected whilst the long narrow stayed mast can be more easily repainted.

The above points are generally covered by a specification issued by the client or commissioning authority, and this comprises tables giving the loads imposed by the lines, so that the designer is not normally required to calculate these loads

If a factor of safety is stated it may be given as a factor on the calculated crippling load on individual members, or, as is usual

with transmission line units, as a factor on the total load sustained before failure on a full scale test of a prototype.

Free Standing Towers.

A typical design is shown in Fig. 1. The tower is tapered in elevation and square at all sections on plan. This diagram shows the various types of bracing in general use.

(1) Z bracing consisting of a single set of diagonals in each face. The bracing members in each panel meet at a common point on each leg. This means that adjacent faces are right and left hand when viewed externally. The bracing takes both tension and compression loads alternately. It

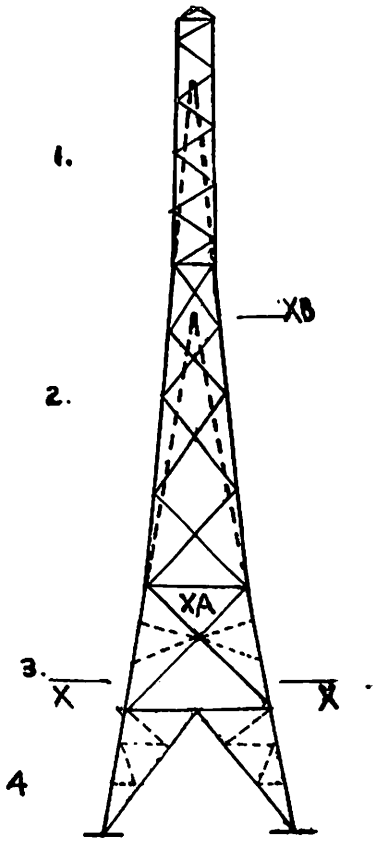


Fig. 1.

is important that the bracing members on adjacent faces meet at a point on any given leg. This is to prevent torsion loads in the structure from being transmitted to the main leg members.

(2) X bracing. Diagonal redundant bracing without horizontals and taking both tensile and compressive loads, cross braces being simultaneously and oppositely loaded. Any face becomes a redundant frame and is designed as such.

(3) Cross bracing with secondary members (shown dotted). These secondary members are only to stiffen the main legs and play no part in taking the shear or torsion loads.

(4) K bracing. Opposite braces loaded tension and compression restrictively. If the load is to the right the right-hand member is compressive, but this condition may be reversed in wide base towers. Secondary bracing is also used here, but it does not stiffen the K brace itself. This may be done by bracing the "hip." The hip is the frame formed by adjacent braces in adjacent sides, and the hip may be diagonally braced as shown in Fig. 2. In small towers, however, single cross members may be used but these should not be placed at the centre of the hip framing but towards the top and bottom of the frame where they will exert the maximum effect, acting as open frame members, to prevent buckling of the main K brace members. The single cross members should have adequate end fixity, that is, two bolt connections as far as possible,

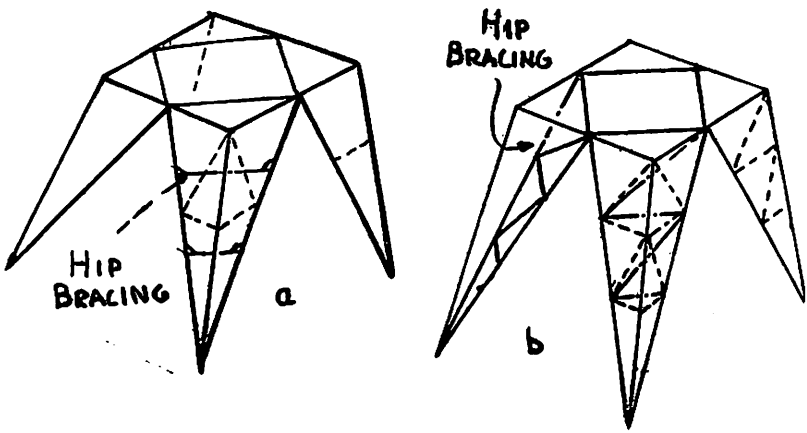


Fig. 2.

or a joint capable of resisting a moment of the order of three per cent. of the main brace load times the distance between the cross members.

If the hip is so large or heavily loaded that this simple form of stiffener is not adequate, full hip bracing is generally resorted to. (Fig. 2 (*b*)).

Uses of Bracing.

Z or X bracing is used where the face width of the structure is such as to allow economic bracing sizes.

K bracing is used in the lower sections, or where the width is too great for the simpler type. The reason for this is that the K brace can easily be supported intermediately against buckling in all directions by means of suitable secondary and hip bracing.

Loads on Tower Structures.

Loads may be divided into the following categories :—

(1) *Verticle Dead and Live Loads.*
These include cable weights including weight of ice on conductors (where applicable).

(2) *Wind Loads on Cables and Conductors.*
Where necessary the cable diameter should be increased by (usually) 1 in. to allow for effect of ice, except, of course, on very steep cables such as stay wires.

(3) *Loads Due to Unbalanced Cable Tensions.*
These occur on transmission towers where there is a change in the direction of the line (transverse load) or where adjacent spans are of different length (longitudinal load), also on radio-masts due to the attachment of aerials, or on the stays of guyed masts under wind conditions.

(4) *Loading Due to Exceptional Conditions.*
This includes broken wire, cable or stay conditions (i.e., emergency conditions).

(5) *Self Weight and Wind Loads.*
It is important to realise, at the outset of a design, the relative importance of this load, as in some cases such as heavily loaded small towers, it has little effect, whilst in the case of tall towers it may represent the whole of the load on the structure, necessitating careful preliminary analysis and one or more trial designs before the magnitude of the load can be assessed.

Load Factors and Factors of Safety.

The factor of safety which is to be used will generally be specified by the client or authority involved, and is normally two on the crippling load of the strut members, such crippling load being determined by formulae which are also generally specified or approved. Typical formulae are given in Appendix 4.

Different factors may be specified for different loading conditions, for instance, all normal maximum loads may be required to have a factor of 2·5, whereas emergency loads only 1·5 is required.

In such a case it is evidently easier to multiply up all loads at the outset, or, if separate stress diagrams are used for the two conditions, as is generally the case, the factors can be brought in before tabulation of the loads. The crippling load in a member should then be equal to the total maximum load in that member determined as above.

Where there are no multiple loading conditions, the design can be carried out in the normal way by dividing the crippling load by the factor of safety being used.

It is to be noted that in either of the above methods care is required in selecting the wind load applicable to each case. (See Table 1, Appendix 5).

In particular cases, such as the stays of guyed masts, a much higher factor may be specified.

Factors as high as 6 on the normal loading may be used in such cases.

Wind Loads.

The wind loading used as a basis of design depends on the geographical area, conditions always being those of maximum exposure. Careful analysis of local conditions should therefore be made to ascertain the wind velocities likely to be encountered. Allowance must then be made for gusts which may produce loadings 2½ times those produced by a steady wind of the same velocity.

Various formulas take account of this extra loading and for structures of low height B.S.S. 449, 1948, may be referred to. (Ref. 3).

The use of the wind formula given therein may be summarised as follows :—

1. Reference is made to tabulated wind loads per square foot of exposed surface.
2. Calculate the projected area of the exposed structural members (that is, the total area projected on to a plane at right angles to the wind direction). The worst case is usually with the wind blowing parallel to a diagonal of the tower.

3. 25% extra stress is allowed over the normal safe working stress in any part of the structure. However, the stresses occurring under windless conditions must not exceed the safe working stresses.

The above formula, although suitable for structures of high solidity, does not give practical results for high towers of lattice construction.

Tests were carried out prior to 1935 to determine the precise loading likely to be encountered by lattice masts and towers due to wind effects. (Ref. 2).

The results may be summarised as follows :—

(1) Wind pressure proportional to V^2 only and not affected by height of structure.

(2) Basic wind pressure $P = 0{\cdot}0042V^2$ where P is in lb. per sq. ft. and V is in miles per hour.

(3) This load is taken on the projected area of the exposed face only and a multiplying factor of 1·5 to 1·8 is used to allow for the other faces. The factor depends upon the plan shape and solidity of the tower.

(4) Wind drag on side faces negligible.

(5) A 12½% increase of load is allowed for a yawing wind.

When designing tall towers the wind velocity is taken as increasing steadily in value from ground level to the top of the tower. The structure may then be divided into sections at each of which the appropriate wind velocity at that height gives a resultant wind load on that section, which is then used for design.

Wind load on circular sections is less than on flat or angle sections and a factor of 0·6 may be used (0·7 where a group of circular sections occurs). As a guide Table I gives the wind loading at various heights for tall towers. (See Appendix 5).

Example.

Cable of transmission tower span 500 ft

Dia. 0·75 in. 52 m.p.h. wind.

Allow ½ in. radial ice thickness = 1·75 in dia.

Wind load = $0{\cdot}0042V^2 = 11{\cdot}4$ lb./sq. ft.

Total wind force

$$= \frac{11{\cdot}4 \times 1{\cdot}75}{12} \times 0{\cdot}6 \times 500$$

$$= \mathbf{450\ lb.}$$

Towers with Heavy Vertical Loads.

Very heavily loaded units may be considered generally to come under the category of building structures. Water towers are good examples of this type, and the plan may conform to the shape of the tank being supported.

Polygonal forms, octagonal or hexagonal are used for circular tanks as shown in Fig. 3, tabular vertical members, with horizontal bracing and cross cable bracing taking tension loads only.

At each panel point internal bracing is used to stiffen the legs in a transverse direction.

The tank itself, of welded plate construction, rests upon radial beams transmitting the load evenly to the main supports.

These radial beams each carry a proportion of the tank load representing a sector of the circular tank and the design is easily carried out by considering a radial and its opposite as a beam spanning across opposite supports and with a double cantilever, each pair of such radials taking a "double wedge" load, maximum at the outer ends and becoming zero in the centre.

The shear due to wind loads may be considered (in the case of a polygonal plan form) as being taken on each face in proportion to its shear resistance.

If shear resistance is denoted by P,

then P is proportional to $\cos \theta$

where θ is the angle made by the face (on plan) with the wind direction.

As an example consider the structure of the water tower shown in Fig. 3. Overall diameter of tank 40 ft. Eight supporting columns of base circle of 27 ft. Tank capacity 15,000 cu. ft., 15 ft. high and 76 ft. from ground. Weight of tank plus contents 420 tons.

Basic wind pressure 28 lb./sq. ft. on tank.

Wind load on tank $= 40 \times 15 \times 28 \times 0{\cdot}6$ lb.

$= 4{\cdot}50$ tons.

$$\text{Vertical load on each col.} = \frac{420}{8} = 52{\cdot}5 \text{ tons.}$$

The wind load on the structure is taken as 4·25 tons (estimated value based on 19 lb./sq. ft. basic wind pressure on structure)

Moment of wind loads about base

$= 4{\cdot}50 \times 81 + 4{\cdot}25 \times 38 = 526$ tons ft.

Denote by I the moment of inertia of the supporting columns (plan section) about XX or YY.

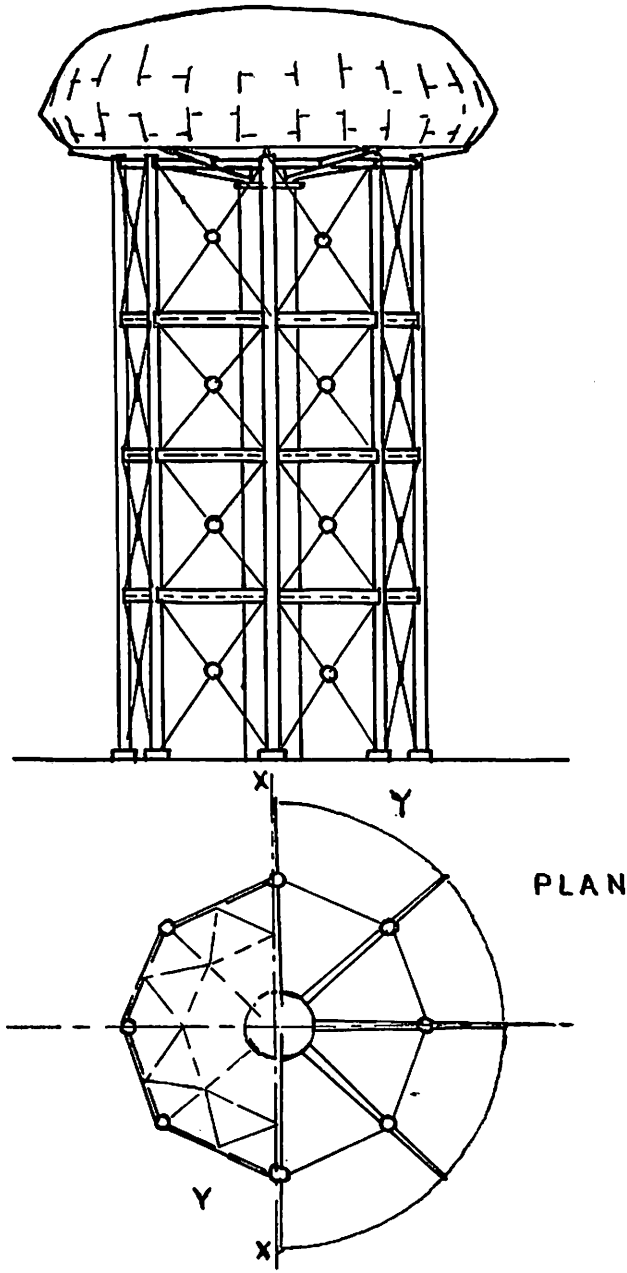


Fig.

$I = \frac{1}{2}$ (polar moment of inertia)

$= \frac{1}{2} \times 8 \times 13{\cdot}5^2 = 728$ units

(assuming each col. = 1 sq. ft. area).

$$\frac{M}{f} = \frac{I}{Y} \quad \text{So } f = \frac{YM}{I} = \frac{13\frac{1}{2} \times 526}{728} = 9{\cdot}75 \text{ tons/sq. ft.}$$

9·75 tons is therefore the column load due to wind overturning moment. As this is less than 25% of the vertical column loads the latter is used for design purposes. If it comes to more than 25% the total direct plus wind load on the column is used for design of the column, and the design stress may then be increased by $\frac{1}{4}$ (B.S.S. 449). Try 10″ dia. tube.

$$l/r \text{ of column} = \frac{13 \times 12}{3{\cdot}70} = 42{\cdot}85.$$

Safe stress = 6·7 tons/sq. in. (from tables)

$$\text{Area required} = \frac{52{\cdot}5}{6{\cdot}7} = 7{\cdot}85 \text{ sq. in.}$$

$$= \textbf{10″ dia. } \times \textbf{ 0.25 in.}$$

The total shear load is 9·75 tons, and the shear distribution on each of the eight faces is proportional to

$1 : 1/\sqrt{2} : 0 : 1/\sqrt{2}$ on both windward and leeward sides, giving a total shear resistance of

$1 + 1 + 4/\sqrt{2} = 4{\cdot}83.$

$$\text{Shear load per face} = \frac{9{\cdot}75}{4{\cdot}83} \text{ tons} = 2{\cdot}02 \text{ tons.}$$

Towers of Considerable Height.

The design is carried out in exactly the same way as for smaller towers, the wind effect, however, being much greater in proportion to wire or aerial loads.

The work involved is therefore much greater as a careful preliminary design must be made to determine the sizes of members and the wind load.

The final design is then embarked upon and when the final wind load is arrived at the necessary adjustments may be made in the original calculations or stress diagrams.

The bracing and leg members will be much larger than in shorter towers, and consequently compound lattice members will have to be resorted to especially in the lower part of the tower where the width may be considerable.

The usual form is triangular or square latticed members for the main legs, and plane lattices for bracing, the lattice being placed

horizontally and needing, therefore, little internal bracing, but some considerable secondary bracing to support it in its weak direction in the plane of the side face.

Resistance of Lattice Structures to Earthquake Shocks.

This may be an important consideration in certain areas, and it may be dealt with by adjusting the base width so that none of the modes of natural vibration of the tower responds exactly to the normal range of frequency of earthquake shocks. Perhaps the most dangerous mode of vibration is with a stationary mode near the top of the tower, and the only method of finding the response of a large tower is by testing a large scale model.

The shock may be considered to travel up the structure in the form of a wave which is reflected from the top, forming stationary waves in the whole structure.

If the length of the wave in the structure is denoted by L, then the distance between stationary modes or points of zero vibration is L/2. The critical heights of structure are then L/2 or 3L/4 or 5L/4, etc., for a square mast and corresponding to the main modes of vibration.

The length of the wave in the tower is given by the formula.

$L = \frac{\text{velocity of propagation}}{\text{vibration frequency}}$ and the velocity of propagation in the structure is proportional to $\sqrt{\frac{\text{elasticity}}{\text{inertia}}}$, the interia is the weight of a small section and the elasticity may be considered as the load divided by the incremental deflection* caused by it in that section.

When scale model testing is carried out the applied frequency is increased to account for scale.

For example with a 1/20 scale model, if the $\frac{\text{load}}{\text{deflection}}$ relation be denoted by K_M for the model and K for the full scale tower and the weights as W_M and W. Then applied frequency for the model would have to be

$$\text{Normal earthquake frequency} \times 20 \times \sqrt{\frac{K_M \times W}{W_M \times K}}$$

Transmission Line Towers.

These may be divided into types as follows :—

(1) Suspension tower (hanging lines only).

* Incremental Deflection means shear deflection per unit height.

(2) Angle tower (sustains side pull due to change of angle of line).

(3) Anchor towers (at the end of a line).

and sub-divided according to the type of line and arrangement of cable anchorages.

(*a*) Single circuit towers (three lines and earth line); top and bottom lines on left-hand extended arm; mid line on right-hand arm.

(*b*) Single circuit bifurcated towers (lines arranged horizontally on long arm). (Example of design given later).

(*c*) Double circuit towers carrying six lines and earth line. Three full cross-arms (design example given later).

Loading.

Apart from wind and dead loads already dealt with, we have in the case of transmission line towers, the line loads which produce a load on each suspension point acting outwards at an angle to the tower since the load is the resultant of the weight of the line and the wind load on it. If the line is attached to the side face (say an aerial), wind load produces torsion on the tower which stresses the bracing in all faces, whilst the weight produces an eccentric load (vertical) which stresses the main legs but not the bracing, unless the tower is tapered, when it affects the side bracing slightly.

It is to be noted, however, that where the line is supported from an extended arm as in the case of transmission towers in general, torsion is not produced by wind load or weight, but by direct pull only as in the case where a line breaks.

As will be seen in the graphical examples, the best way of dealing with this complex loading is based on the fact that any line attachment gives rise to an inclined load on the tower (the resultant of the vertical and horizontal loads) and possibly a torsion load.

The inclined load is easily brought into the stress diagram on the faces parallel to its own plane, whilst the torsion load is added to the stress diagram of each face in turn.

Load and Safety Factors.

If, as is common, there is a different factor for the torsion loads as distinct from the other loads, they must be factored first and then all loads may be incorporated on the same stress diagram. In this respect it is often easier to design on ultimate breaking loads, that is safe loads multiplied by the appropriate load factor. (See Appendices 2 and 3).

Detail Design by Calculation.

Method of Sections.

Suppose the side wind loads are applied (Fig. 1) at the panel points (*i.e.*, the distributed wind load is reduced to panel point loads). Imagine the tower cut at XX. The section above is in equilibrium from the side forces applied externally and the structural members (two legs and two braces) below XX and cut by it.

To determine the loads in the K braces take the sum of the moments of the external loads about XB and divide this sum by the total lever arms of the brace members about XB.

(The lever arm is the distance from XB to the line of the brace produced and normal to this line).

To find the leg loads on the line XX take external moments about XA and divide by the sum of the distances from XA to the main legs and normal to them.

The results are the brace and leg loads respectively and the opposite leg or brace carries the same load of opposite sign. (Vertical loads are ignored at this stage).

A similar method is used for single bracing members and the type of loading (T or C) may be determined from the direction of the main external loads. For any given loading system the load in the brace is always considered compressive if the loads can be reversed.

It may be easily seen from the above that any brace load is zero if the main leg members at that level converge to a point, which coincides with the centroid of the transverse loads above it. In practice this cannot occur because of the variable nature of some of the loads, and the effect of the vertical ones, but it gives a good guide to the most efficient shape for a given design, and is therefore a useful principle in preliminary design work.

Eccentric Vertical Loads.

The tower of Fig. 4 is eccentrically loaded due to the staggered support arms. Basing calculations on the principles of the last section, take moments about XA to obtain the moments of all loads above section YY. Again all loads below this section are ignored

Moments of horizontal forces

$$= 1 \times 70 + 1 \times 60 + 1 \times 50 + \tfrac{1}{2} \times 40 = 200 \text{ tons/ft.}$$

Moments of vertical forces

$$= +2 \times 10 - 2 \times 10 - 2 \times 10 = -20 \text{ tons/ft.}$$

$$\text{Total} = 200 - 20 = \mathbf{180} \text{ tons/ft.}$$

Produce *ca* and take normal distance to this line from YA

$$= 53 \text{ ft.}$$

$$\text{Brace load} = \frac{180}{53} \times \frac{1}{2} \times \frac{1}{2} = 0{\cdot}85 \text{ tons.}$$

Note the division in all by four as there are two side faces and two bracing members cutting the section YY.

If there were only one full diagonal brace cutting YY, the division would be by two, *e.g.*, for a single brace at YY the lever arm is 67 ft., so brace load (Z brace) $= \frac{180}{67} \times \frac{1}{2} = 1{\cdot}34$ tons.

That *ab* is compressive and *ac* tension is easily seen by considering the direction of the moment of the external loads about YA (anti-clockwise).

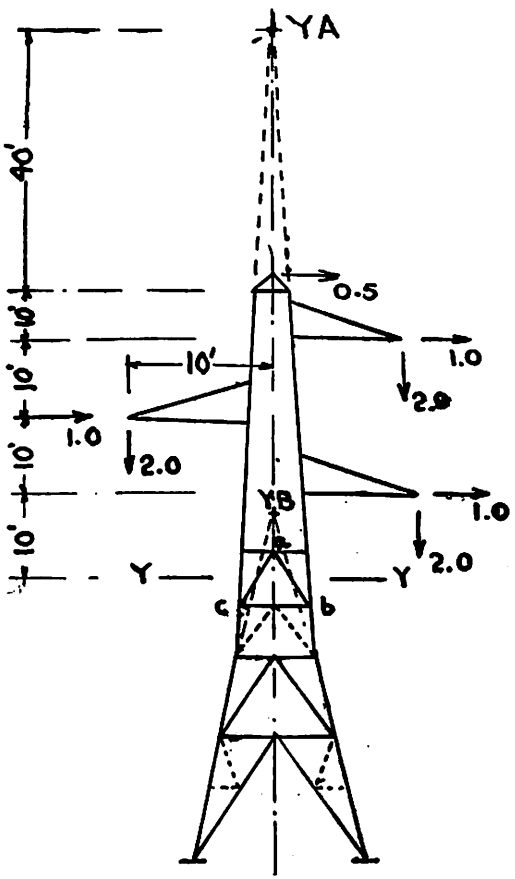


Fig. 4

Similar sections may be taken in each panel and all brace loads found. When YY is in the lower section of the tower take moments about YB.

By using a sign convention for the moments (+) for anticlockwise (−) for clockwise, for instance, confusion is avoided (as in the case of moments about YB both signs exist).

Leg Loads.

Take moments about a

$1 \times 10 + 1 \times 20 + 1 \times 30 = 60$ horizontal.

$2 \times 10 + 2 \times 10 - 2 \times 10 = 20$ vertical.

Total 80 tons/ft.

Lever arm of leg load about $a = 4$ ft.

$$\text{Leg load} = \frac{80}{4} \times \frac{1}{2} \times \frac{1}{2} = 5 \text{ tons.}$$

To this is added the vertical load distributed over the four legs, *i.e.*, $3 \times 2 \times \frac{1}{4} = 1\frac{1}{2}$ tons.

Leg loads—right $+5 + 1\frac{1}{2} = +6\frac{1}{2}$ tons.

left $-5 + 1\frac{1}{2} = -3\frac{1}{2}$ tons.

In the case of one single brace, take moments about the intersection of the brace with the main leg above YY. The moment is 80 tons/ft. as before.

$$\text{Lever arm} = 8 \text{ ft.} \quad \text{Leg load} = \frac{80}{8} \times \frac{1}{2} = 5 \text{ tons.}$$

Torsion Loads.

As already explained, any load acting longitudinally on one face produces torsion in the structure. Torsion is also caused by unbalanced forces acting parallel to the line on the cross arm ends of a transmission tower.

Consider an example of the latter load as shown in the plan view of a cross arm, Fig. 4 (*a*).

To analyse the loads in the members of the cross-arm and the tower itself, we may proceed as follows :—

Replace the 1 ton offset load with a central horizontal load P combined with a clockwise moment Px. The load P is equal to 1 ton, and is resisted by the side faces only.

The effect of the moment Px has now to be considered. If this is equally distributed among the four faces, the force on each is $\frac{Px}{4 \times b/2}$ or $\frac{Px}{2b}$ where b is the breadth or plan length of one face at the point of application of the load.

In arriving at this result two assumptions are made :—

(1) The tower is square on plan.

(2) The section remains square after torsional deflection.

Condition (2) means that the tower must be rigidly cross braced at all points of application of a torsion load and at points on elevation where an abrupt change of profile takes place.

Subject to these conditions the faces may be stressed by graphical or other means for loads applied in the above manner as follows :—

Front face $\frac{Px}{2b}$

Side face $\frac{Px}{2b} + \frac{P}{2}$

where P is the load and x the distance of its line of action from the centre.

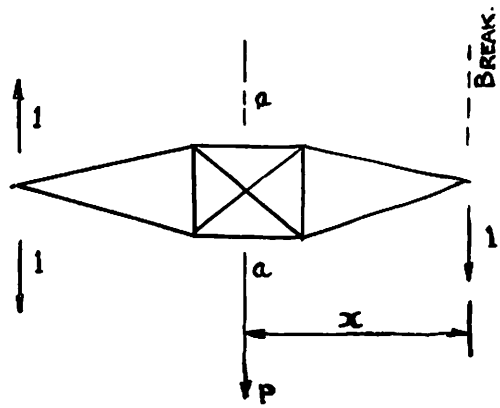


Fig. 4 (a).

Rectangular Plan Form—(Fig. 5).

The second condition is still assumed to hold in this case, that is, that cross bracing maintains the plan form rectangular. By drawing out the plan form in an arbitrary deflected position, Fig 5, it is evident that the greatest deflection takes place on the shorter side and that the deflection in the direction of one face is inversely proportional to its breadth.

The deflection referred to is, of course, the shear deflection of the face which in a lattice braced structure is proportional to the load causing it, irrespective of the breadth. Consequently, since the deflection is inversely proportional to the face width, so must be the load, and the torsion may therefore be considered as distributed among the faces in inverse production to b and d.

Referring to Fig. 5.

Let torsion $= Px$ as before.

Faces of breadth b and d.

Denote by T_d the torsion resistance of face d, that is the load in that face, and by T_b the load in face b.

$$\text{But } T_d = T_b \frac{b}{d}$$

$$\text{and } \left(T_d \times \frac{b}{2} + T_b \times \frac{d}{2}\right) = \frac{Px}{2}$$

$$\text{so } \left(T_b \frac{b^2}{2d} + T_b \frac{d}{2}\right) = \frac{Px}{2}$$

$$T_b \frac{b^2+d^2}{d} = Px$$

$$T_b = Px \frac{d}{b^2+d^2}$$

$$T_d = Px \frac{b}{b^2+d^2}$$

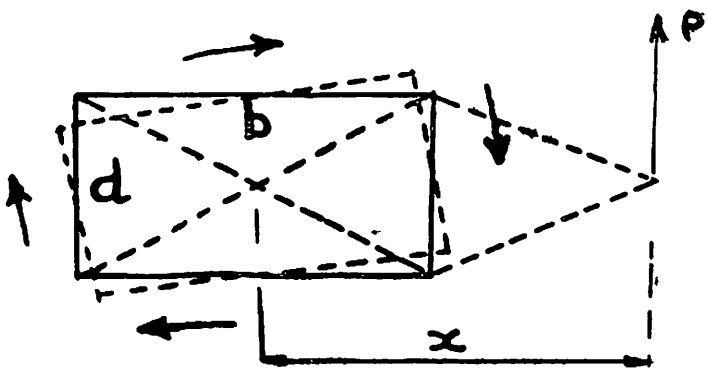


Fig. 5.

As in the previous case the value P/2 is added to each of the side faces and calculate the front and side faces for horizontal co-planar loads of the above value.

Example.

The section X – X in Fig. 6 is to be calculated for leg and brace loads as a result of the vertical and horizontal loads shown plus a torsional load introduced by a broken wire on the end of the top cross-arm of 1 ton. All loads are given in tons.

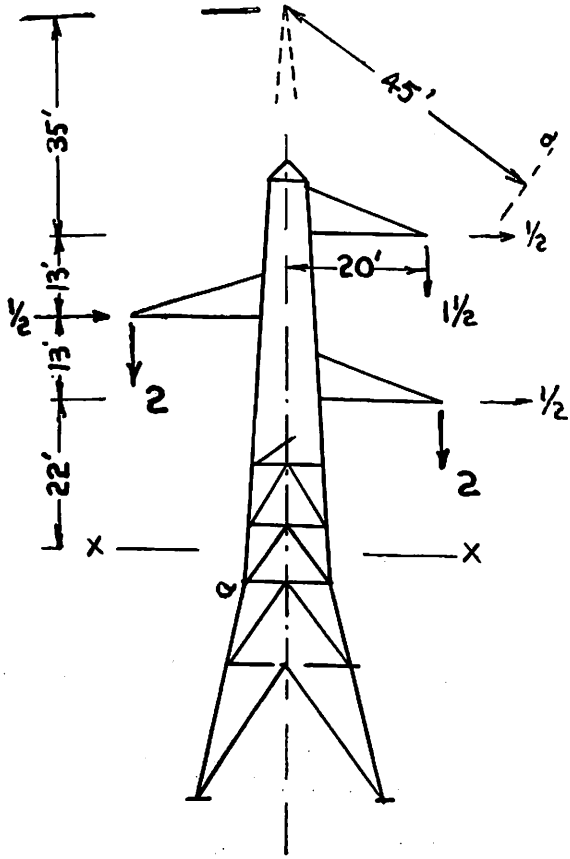


Fig. 6.

(1) *Brace Loads.*

(*a*) Transverse and eccentric loads front face. Anticlockwise moments about meeting point of legs.

$\frac{1}{2}(35+48+61)+2\times20-2\times20-1\frac{1}{2}\times20 = 42$ tons/ft.

$$\text{Brace load} = \frac{1}{2}\times\frac{42}{45} = 0{\cdot}46 \text{ tons.}$$

$$= \textbf{0·23 tons per face.}$$

(*b*) Torsion side face direct load.

$$1\times35 = 2Q\times45.$$

$$Q = \textbf{0·39 tons.}$$

$$= \textbf{0·195 tons per face.}$$

(*c*) Torsion equivalent load on each face.

$$\frac{Px}{2b} = \frac{1\times20}{2\times4} \text{ (width of face at this level} = 4 \text{ ft.)}$$

$$= 2{\cdot}5 \text{ tons.}$$

$$2{\cdot}5\times35 = 2Q\times45.$$

$$Q = 0{\cdot}96 \text{ tons.}$$

$$= \textbf{0·48 tons per face.}$$

(2) *Right-hand Leg Load.*

Taking moments about intersection of braces.

$$-\tfrac{1}{2}(48+35+22) - 20(1\tfrac{1}{2}+2) + 20\times2 = -82{\cdot}5 \text{ tons/ft.}$$

$$\text{Leg load} = \frac{82{\cdot}5}{6} = 13{\cdot}75 \text{ tons.}$$

$$\text{Per leg} = \textbf{6·88 tons.}$$

In addition to this load which is tension one side and compression the other, there are the vertical loads totalling 5·5 tons or

1·38 tons per leg.

and the side face torsion load $\frac{1\times48}{6} = 8$ tons

$$= \textbf{4 tons per leg.}$$

The loads may be tabulated as follows :—

Brace	Front Face	Torsion	Side Face	Total	Max.
XX	0·23	0·48		0·71	0·71
		0·48	0·195	0·68	

Leg	From Vert. Lds.	From Hor. Lds.	T.	C.	Tors.	Max. T.	Max. C.
XX	1·38	6·88	5·50	8·26	4·00	9·50	12·26
			Col. (3−2)	(3+2)		(4+6)	(5+6)

The last column is the design load, the seventh column in Table 2 being used only for leg joint design if there are bolts in tension.

It is to be noted that when calculating the leg load due to torsion the quantity $\frac{Px}{2b}$ (torsion load on each face) is not considered, but the equivalent load acting at the centre of the tower only is taken. This is because in a square section, the torsion load only effects the bracing, the leg loads cancelling out. In a rectangular section, however, the effect of the torsion distribution (which is unequal) must be considered, and in this case work out the leg load due to this torsion on the wide face and the narrow face and take the difference.

When using graphical methods the tables are made up from the loads scaled from the stress diagram, but in this case the leg loads T and C are scaled direct and only the last five columns appear in the second table. (See Appendices 2 and 3).

Internal Cross Bracing.

By analysing the loads in a single cross brace such as one of the pair shown in Fig. 5, it may be seen that the load W in the internal member due to a load P acting horizontally distance x from the centre is given by :—

$$W = Px \frac{d}{b} - P \frac{b^2+d^2}{2b}$$

If the section is square $b = d$, and W becomes $= P(x-b)$.

If the bracing is double (crossing in the centre) one half of the value of W may be taken for each.

Example.

1 ton at 15 ft. 4 × 4 ft. tower section.

Load $= P(x-b) = 1(15-4) = 11.$

Load per brace = **5·6 tons.**

If the section is 4′ × 6′

$$\text{Load} = 1 \times \frac{15 \times 6}{4} - 1 \frac{16+36}{8}$$

$$= 16.$$

Load/brace = **8 tons.**

Practical Design Consideration.

It may be found difficult to decide upon the exact profile of the main legs and the best bracing arrangement to use. As described previously, having produced a preliminary design and calculated the loading, the main leg lines should meet (when produced) at the centroid of the loads above it. However, a compromise is usually adopted, as base widths are sometimes limited in size, and also, near the top of the tower the principle cannot be applied as the legs are parallel, or nearly so. At this level the width generally to be adopted is that giving smallest number of brace members consistent with reasonable member size. Stress reversal, that is, a member normally to be expected in tension is found to be in compression, may occur in wide base towers, and is a sign that the base width may be effectively reduced. (See Fig. 15).

Strut Members.

Complete end fixity such as may be considered to occur in building design does not occur with the bracing members of towers. This end fixing depends just as much upon the torsional resistance of the leg members and the fixity produced by the attachments on adjacent faces as upon the end connections of the brace itself. In small towers, therefore, single bolt connections are the rule rather than the exception, and tubular or box members used for the main legs (or box lattice sections on high towers) increase the efficiency of the bracing members as well as allowing greater unsupported leg lengths.

Example of Compound Strut.

A strut consists of two steel members of 2 sq. ins. cross sectional area (a) radius of gyration (g) = 1·25", set 9" apart and suitably braced. The length is 25 ft. and the strut is supported by two equidistant secondaries normal to its plane. Calculate the maximum load assuming a Q factor of 0·85.

Weak Axis.

l = 25/3 × 0·85 = 7·09 ft. (85 in.)

g = 1·25 in. l/g = **68.**

Safe stress by Johnson's formula (mild steel)

= 15/2 (0·737) = 5·52 tons/sq. in.

Safe load = 5·52 × 2 × 2 = **22·08 tons.**

Strong Axis.

l = 25 × 0·85 = 21·25 ft. (256 ins.)

$I = 2 \times 2 \times (4\frac{1}{2})^2 + 2 \times 1{\cdot}25^2 \times 2 = 87{\cdot}24$ in.^{4}

$$g = \sqrt{\frac{I}{A}} = \sqrt{\frac{87{\cdot}24}{4}} = 4{\cdot}67 \text{ in.}$$

$l/g = 55$. Safe stress = 6·0 tons/sq. in.

Safe load = **24 tons.**

All members in side faces take both tension and compression as loads are generally assumed to be reversible (as will be seen later this does not necessarily apply to redundant bracing). In this case, of course, the tension and compression are equal and the member is designed as a strut, the member being checked to ensure that the tensile load can be carried in the area around the connecting bolt.

If the structure carries a load such as an aerial, on one side, the above does not apply to the stresses arising from this load. This part of the design work must therefore be kept separate and a careful check kept on whether the load produced in a member is tension or compression. In most cases where the loads are reversible, such as in the case of a transmission line tower or free standing tower, the members are designed for the maximum total load and is taken to be compressive.

For the structural analysis of a face, whether front or side, that face is considered to be plane and is treated as a plane frame even if one or more changes of profile take place. The only loads not appearing in this analysis and which have to be considered in the actual structure are the internal cross brace loads. Cross bracing is usually incorporated at each cross-arm level and often at any change of profile as well.

Fig. 7 shows some general forms of internal cross bracing in common use.

As previously noted the cross bracing maintains the rectangular form of the tower under torsion loads, which are distributed by this bracing among the faces in inverse proportion to their widths.

If, however, the section of the tower changes, for instance from rectangular to square, the distribution will also change, but only if cross bracing is introduced and only at the points where it occurs. If the cross section is changing in shape, therefore, the torsion must be recalculated at each cross bracing point. (See Appendix 1).

Each face is analysed completely by either the method of sections or graphically (see Appendix 2), and it is generally convenient not to separate wind and dead loads but to take the resultant of these at each load point. (This should be clear from the stress diagrams given—Appendix 2).

If, however, a load has a different effect on the structure from the others, it must be dealt with separately and a complete analysis made or a stress diagram drawn.

As a simple example consider a square lattice tower with a single cross-arm carrying two overhead lines, one each side, and freely suspended.

The maximum normal load on this T shaped tower would be a load on each end of the T inclined downwards and to the right. A single stress diagram will give the member loads in the front (and rear) face. Due, however, to a broken line on one side there is a possibility of a torsion load affecting the bracing of all faces and a longitudinal load on the side faces.

There are therefore three separate stages in design of this unit :

(1) Front face vertical and side wind loads combined.

(2) All faces (bracing only) torsion load (take equivalent load on each face).

(3) Side face. Longitudinal load due to one half of the torsion load on each side face.

If the tower is tapered on elevation instead of being of constant width, exactly the same procedure is followed. It may be thought that in this case it is not correct to take equivalent loads on each face for the torsion load owing to the greater torsion resistance of the structure at the base. It is not, however, necessary to consider this at all, if the methods previously described are used for analysis. For instance, if the stress diagram is drawn in the normal way for the equivalent face load at the level at which torsion is applied ; the brace load scaled from this diagram at any point (say the base)

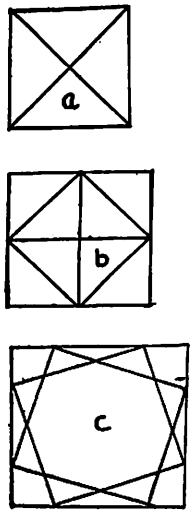


Fig. 7.

is equal to the load obtained by calculating the torsion resistance at that point. (A proof of this is given in Appendix 1).

The face is considered as being plane from top to bottom. With normally tapered structures it is not necessary to make any further allowance for the angularity of the legs after completing the stress diagram, the horizontal loads occurring at a profile change point already being incorporated in the diagram. If, however, the angularity of the main legs becomes very great the main leg loads only should be multiplied by $\frac{1}{\cos \theta}$ where θ is the angle made with the vertical. The horizontal cross brace at the change point also takes an additional load equal to leg load $\times \tan \theta$.

Redundant Members.

If a stress diagram can be drawn for any lattice frame, there are no redundancies and it is said to be statically determinate, that is, the member loads may be determined from the external conditions. Statically indeterminate structures cannot be so analysed and may have either more external supports than are required for stability (a condition hardly ever occurring with towers) or more internal members. It may be wondered why these extra struts or ties are put in at all; they do, however, provide extra support for legs, and take their share of the load. As an example Fig. 1 shows redundant bracing at **2** and **3** but not **1** and **4**. Figs. 19, 21 have redundancies shown dotted. In contrast to plain bracing in which all members take compression at some time or other, redundant bracing systems always have tension members whichever way the load is applied. This must never be overlooked when designing redundant frames as great economies in material can usually be made.

Before attempting any graphical analysis, all redundant members must be removed. This has been done in all the examples quoted and the structure analysed as a "just stiff" framework. At this point re-inserting the redundant members (which take some of the load and become stressed) reduces the loads so that the actual loads are less than these measured. The precise calculation of the proportion of the load taken by the extra struts and ties is complex and only a brief description is given here. (See reference 12 for details of various methods). It must be realized, however, that where there are a large number of redundancies as in the case of the tower—Fig. 21, for instance, the working takes a great deal of time and the result is little different from a simple division of the member loads. In some simple frameworks, however, simply dividing the load between the redundant and non-redundant parts does not give an accurate result, and in any case the accurate

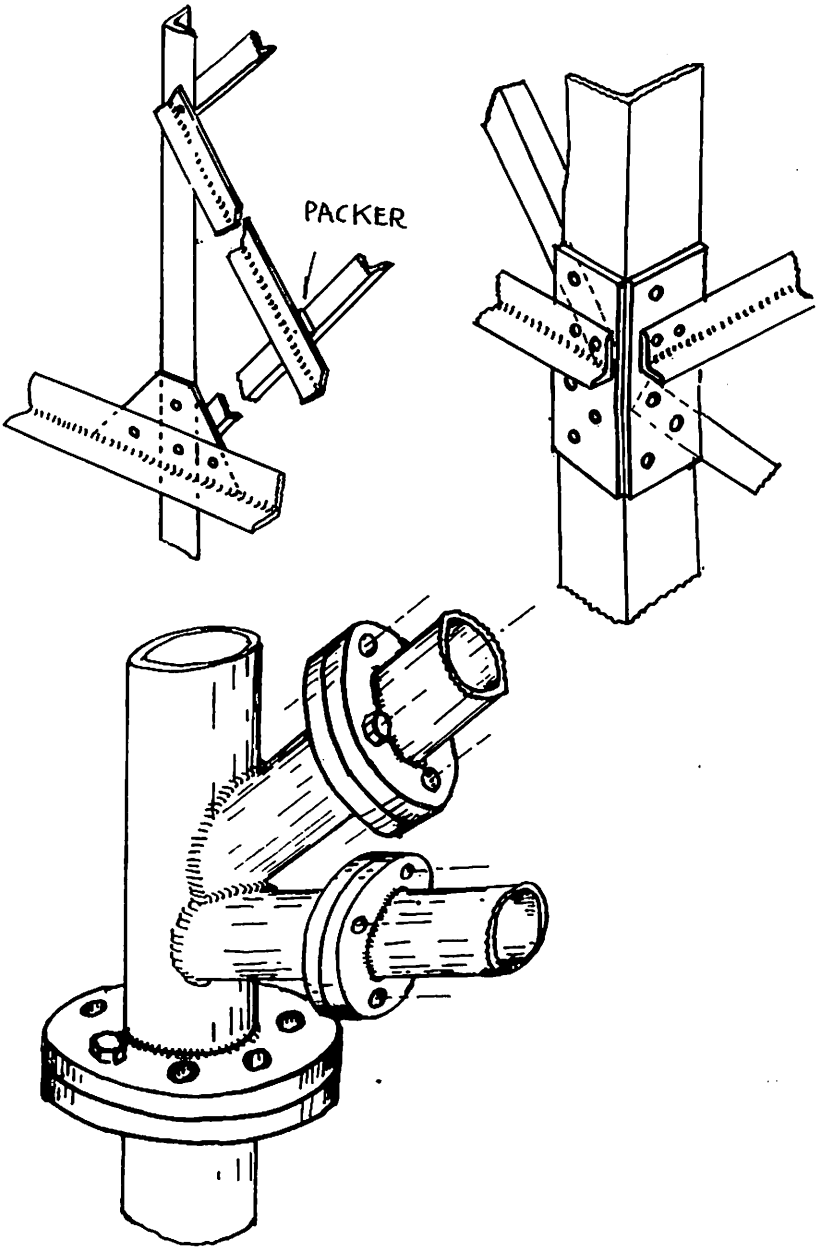


Fig. 8.

method would not be so complex. The procedure is briefly as follows :—

(1) Remove all redundancies. The remaining structure is such that it is just rigid and a stress diagram can be drawn from it. The choice of which members are redundant is in most cases immaterial as long as the remainder is not deficient. If a stress diagram can be drawn it is neither deficient nor redundant.

(2) Each of the original redundant members is replaced by two equal and opposite loads acting at its extremities. Call these R_1, R_2 . . . . etc.

(3) Make a stress diagram for the remaining structure with its external loading, and one for each of the imaginary loads R_1, R_2, etc., without the external loads, the R's being taken as unit loads.

(4) For each member of the unit compute $\frac{PL}{AE}\,a$, $\frac{PL}{AE}\,b$, $\frac{PL}{AE}\,c$, where L is the length and A the cross sectional area of the member, P the load due to the external loading combined with those due to $R_1 + R_2 + R_3$, etc., and a is the ratio $\frac{\text{member load due to } R_1}{R_1}$, b is the ratio $\frac{\text{member load due to } R_2}{R_2}$ and so on. Each expression is summed for all the members and each sum put equal to zero. On solving the simultaneous equations so formed, R_1, R_2, etc., may be found, and by factoring the stress diagrams for these values the whole of the member loads throughout the frame may be found.

The work must be tabulated for best results. (See Ref. 12).

GUYED MASTS.

Structural Principles.

In contrast to the free standing mast in which the overturning moment due to wind and aerial tension loads is carried to the bottom of the structure and transferred to the ground by tension and compression in the leg members, the guyed or stayed mast is directly supported by means of the stay wires as far as the transverse loads are concerned, but the stays, due to their tension, exert a vertical

force on the mast. The guyed mast, therefore, has negligible bending moment at the base (in fact the base is often pin jointed), but a high vertical compression.

The mast itself is usually a latticed braced member of narrow triangular cross section which is of constant width, in this case three stays being used at each level at which stays are attached. This is for convenience only, four stays being equally usable, but naturally less efficient from the point of view of material usage.

The construction of the mast itself lends itself admirably to welding and the mast may be shop fabricated in sections in the form of lattice tubular or angle units in lengths of 20 to 40 ft. needing the minimum of bolting on site.

Under wind load the mast acts as a continuous beam between stay attachment points at which the reactions occur, so that the stress distribution in the mast and the compression loads therein, when the stay tensions are known, are easily calculated.

Before describing the method of calculating the stay tensions it is necessary to explain fully the functions of the stays under various loading conditions. The stays are normally set at angles of between 30° and 50° with the ground and may be of high tensile steel cable suitably protected against corrosion by an aluminium or zinc coating. Stays are very inefficient at angles exceeding 60°, and most masts in service have stays at much more shallow angles.

The number of stays varies greatly in masts already built. Masts of 800 ft. with only four sets of stays are in use, but in this country the general practice is to use up to six sets for masts of 500 ft., the three bottom sets having common anchorage points set on lines mutually at 120° radiating from the mast base, and three further bases about 300 ft. out and common to the three top sets of stays.

Continental practice, however, favours fewer stays and a stouter mast section. One radio mast is over 1000 ft. in height and has a maximum section of 24 ft. square, but only one set of stays of 2½ inches diameter high tensile cable. (See Fig. 10 for a diagram of a similar mast).

It is seen from this that the selection of stay positions is not rigidly determined in the structural design but is the result of many factors. Cost of mast, local conditions, ground area available, and the electrical characteristics of the mast itself (where this is used to carry electrical currents as in some cases where used for broadcasting) are some of the main considerations.

Safety Factors.

The normal working loads in the members of the mast structure will not normally exceed about a third of the crippling load, if the

design is correct, but all components of the load including local bending, etc., must be taken into account. In some instances higher factors may be specified, or a load factor based upon ultimate loads rather than normal working loads. The stay wires are subject to much higher factors, the normal safe working load on high tensile stranded cable being usually one fifth or one sixth of the breaking load, depending upon type.

Stay Tensions.

In small masts, the tensions in the stays produced by wind on the mast may be easily calculated by resolving the mast wind reaction at the stay attachment points. By adding the initial tension required to make the stays taut under no wind conditions, and resolving in the vertical and horizontal directions, the load on the mast and drag and uplift on foundations are easily determined. In the case of very high masts, however, the method of determining stresses is more protracted.

This is because the stays cannot be considered as taut cables, even with high initial tensions, and they sag under their own weight and the wind load. The greater the wind and dead load and the less the tension, the greater the sag will be. The cable also undergoes elastic extension under this tension and both the lengthening due to this and the shortening due to sag must be taken into consideration.

The initial tension in the stays, that is, the tension under zero wind load, determines (in any given mast) the maximum deflection under full load.

This initial tension or pre-tension is always used and the higher it is the less will be the movement of the mast top under ultimate wind load.

Because of the usual form of pin jointed base, the allowable deflections on guyed masts are small, although any effort to decrease them unduly by increase of stay pre-tension results in a heavier stay cable and a high vertical loading in the mast, which is not always permissible from a structural point of view.

Wind Load.

The design wind loads will in most cases be determined by local conditions. Normal European ultimate design loads may be taken as given in Table I (Appendix 5). These are ultimate loads and if the design is based upon safe stresses in lieu of ultimate stresses, half the given wind loads may generally be taken.

The loads are calculated for one face and the other faces are taken account of by using a multiplying factor which may vary from 1·4 to 1·8, depending upon the shape and solidity of the mast.

For circular sections such as stay cables a factor of 0·6 may be used (but 0·7 for circular mast sections where latticed).

In practice, the wind load at each stay attachment point is estimated and, assuming full wind load, the leeward stay tension is given an assumed value. From this, the expected tension on the windward side is calculated, and a deflection calculation is made. If this calculated deflection is different from the maximum allowable deflection of the mast, the difference is adjusted by taking a new leeward stay tension which shortens or lengthens the stay chord by an amount corresponding to this difference, and the windward stay tension is re-calculated. (For an equivalent graphical method see Reference 5).

Changes in Stay Length.

Suppose a stay wire is spanning between the ground A and the attachment point B (Fig. 10). AB = L ft, and the actual length of the strained cable measured along the curve is l ft. Suppose

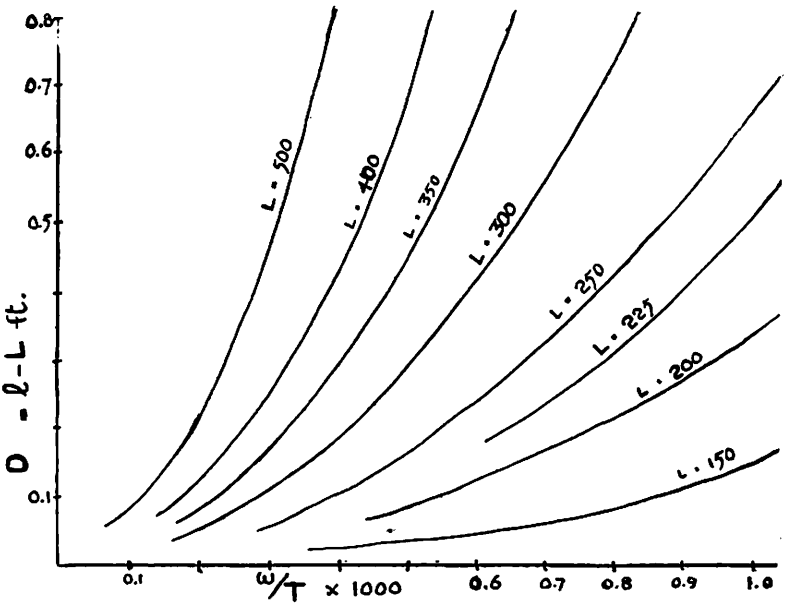


Fig. 9.

a sag d ft. is produced by a tension of T lb. The loads due to the wind on the cable and the weight of the cable itself are combined, with full consideration to the different directions of the loads. (See Fig. 12 and Appendix 7). The resultant is now resolved into components along and normal to the cable. The former is ignored. Let the latter be denoted by w lb. per ft. run.

By taking moments about O (Fig. 11)

$$\frac{wl^2}{8} = Td$$

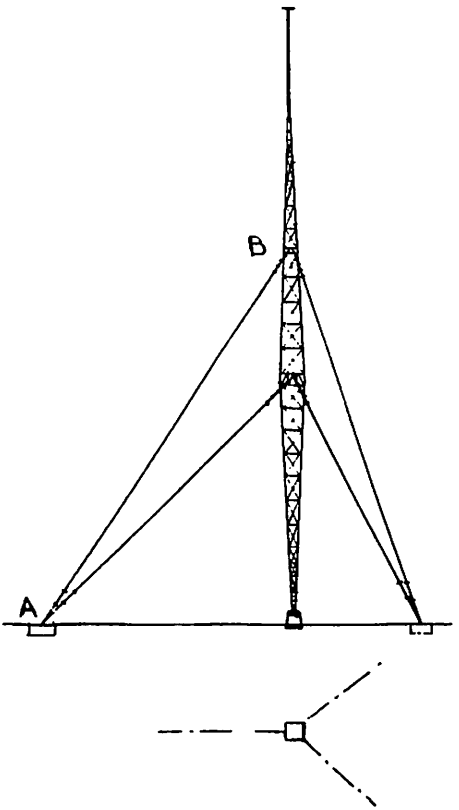


Fig. 10.

$$\text{or sag } d = \frac{wl^2}{8T} \quad (1)$$

As the sag is small, the curve followed by the cable may be considered as parabolic, for which curve the length minus chord can be shown to be :—

$$l - L = \frac{8d^2}{3l}$$

Substituting for d from (1) above

$$l - L = \frac{w^2 l^3}{24T^2} = \text{(nearly) } \frac{w^2 L^3}{24T^2} \text{ or } \frac{L^3}{24}\left(\frac{w}{T}\right)^2$$

The plot of $l - L$ against w/T is shown in Fig. 9 for various values of L.

The elastic extension of a cable of cross section a sq. in. $= e$ ft. $= \frac{lT}{Ea} = \text{(nearly) } \frac{LT}{Ea}$. E is Young's modulus and is taken as 30×10^6 lb/sq. in. for convenience and clearness. This should be carefully checked in individual cases as stranded wires have different elastic properties from solid sections (see Reference 10). For instance, if E refers to the material of the cable, then a is the nett area of the wires. In practice, however, it is essential to take a as the gross area of the cable as stranded in which case E is to be the apparent modulus.

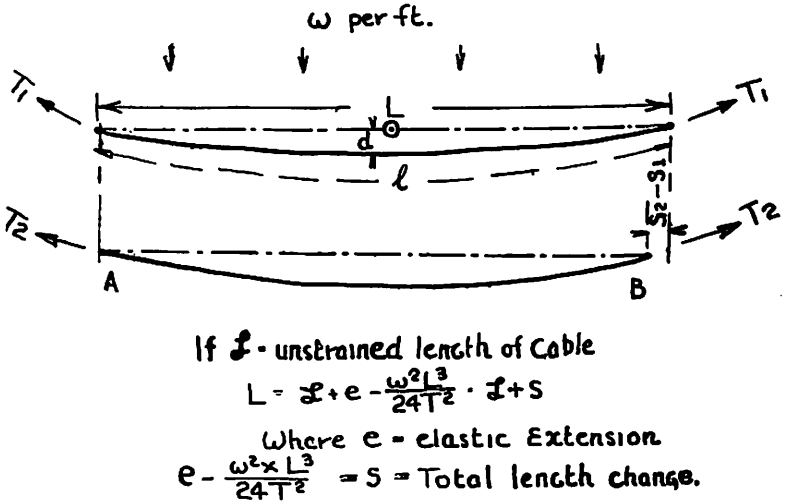


Fig. 11.

Referring again to Fig. 11, the properties of a sagging cable may be determined as follows :—

The geometrical difference in length between the chord L and the strained length l of the cable $l - L$ is denoted by D.

The difference $\frac{LT}{Ea} - \frac{w^2L^3}{24T^2} = e - D$, may be denoted by S, and S is the difference between the length of the cable itself and the length of the chord of the stay wire curve when loaded with tension T and transverse load w per ft.

As the stay wires are originally the same length and height, by calculating S for the leeward and windward stays, the difference is equal to the change in stay lengths as calculated from the deflection of the mast. An example will make this clear.

Four guyed mast 200 ft. high ; 4,000 lb. at top normal to mast. Guy angle 63°, wind 30 lb./sq. ft. Stay characteristics may be taken from the following table :—

Stay Dia.	a sq. in.	lb. load	wt./ft.	Wind area/ft.	Stretch/1,000 lb./1,000 ft.
0·475	0·196	7,900	0·43	0·039	0·17
0·625	0·307	10,350	0·73	0·052	0·10
0·70	0·380	12,825	0·93	0·060	0·09

L = 228 ft. (200/sin 63°).

Difference in stay tension $= \frac{4{,}000}{\cos 63} = 8{,}800$ lb.

Assume deflection at top to be ½% or 12 in.

Shortening of leeward stay = lengthening of windward stay

$= \frac{12 \cos 63}{2} = 2{\cdot}75$ in. $= 0{\cdot}23$ ft.

Total change of stay length under load = 0·46 ft.

As the minimum stay tension will lie between 8,000 lb. and 11,000 lb., a 0·625 cable will probably be required.

Component of wind normal to stay $= 30 \times 0{\cdot}052 \sin 63° \times 0{\cdot}7$

$= 0{\cdot}97$ lb./ft.

Component of stay weight normal to stay

$= 0{\cdot}73 \cos 63° = 0{\cdot}33$ lb./ft.

w (no wind) $= w_1 = 0{\cdot}33$ lb.

w (leeward stay) $= w_2 = 0{\cdot}33 - 0{\cdot}97 = -0{\cdot}64$ lb.

w (windward stay) $= w_3 = 0{\cdot}33 + 0{\cdot}97 = 1{\cdot}30$ lb.

As a first approximation take leeward stay tension T_1 as 1000 lb.
Then T_2 (windward stay) = 1,000 + 8,800 = 9,800 lb.

Windward Stay.

$$\frac{w_3^2 L^3}{24 T_2^2} = \frac{1{\cdot}3^2 \times 228^3}{24 \times 9800^2} = 0{\cdot}0087 \text{ ft.}$$

$$e = 0{\cdot}1 \times \frac{228}{1000} \times 9{\cdot}8 = 0{\cdot}223 \text{ ft. (0·1 from table)}$$

$$S_1 = 0{\cdot}223 - 0{\cdot}0087 = 0{\cdot}214 \text{ ft.}$$

Leeward Stay.

$$\frac{w_2^2 L^3}{24 T_1^2} = \frac{0{\cdot}64^2 \times 228^3}{24 \times 1000^2} = 0{\cdot}203 \text{ ft.}$$

$$e = 0{\cdot}1 \times \frac{228}{1000} \times 1 = 0{\cdot}0228 \text{ ft.}$$

$$S_2 = 0{\cdot}0228 - 0{\cdot}203 = -0{\cdot}18 \text{ ft.}$$

Total change in length = 0·214 − (−0·18) = 0·394 ft.

This is less than the 0·46 ft. allowed, so a reduced value of T_1 is taken for a second approximation. The difference is 0·46 − 0·394 = 0·066 ft.

Leeward Stay.

$$\frac{w_2^2 L^3}{24 T_1^2} = 0{\cdot}203 + 0{\cdot}066 = 0{\cdot}27 \text{ ft.}$$

from which T_1 = 870 lb.

and e = 0·02 ft. by re-calculation.

and S_2 = 0·02 − 0·27 = −0·25 ft.

Windward Stay.

T_2 now equals 870 + 8800 = 9670 lb.

By calculation of the expression $\frac{w_3^2 L^3}{24 T_2^2}$ using the value 9,670 for T_2 instead of 9,800, the value 0·009 ft. is obtained for $l - L$.

e now becomes 0·22 instead of 0·223

and S_1 = 0·22 − 0·009 = 0·211

$S_1 - S_2$ = 0·461 ft.

This second approximation is correct, and it is now only necessary to calculate the pretension T. When the wind load is

removed and the mast brought upright the leeward stay lengthens by $\frac{0{\cdot}46}{2} = 0{\cdot}23$ ft.

So S (leeward stay) $= -0{\cdot}25 + 0{\cdot}23 = -0{\cdot}02$ ft.

$$e - \frac{w_1^2 L^3}{24T^2} = -0{\cdot}02 \text{ ft.}$$

Putting T in thousands of lbs.

$$0{\cdot}1 \times \frac{228}{1000} \times T - \frac{0{\cdot}33^2 \times 2{\cdot}28^3}{24T^2} = -0{\cdot}02.$$

$$T = \mathbf{1{,}110\ lb.}$$

Masts with Multiple Stays.

The stay wires are generally placed at equal distances up the mast and each set of stays in the same vertical plane.

The maximum angle of the highest stay with the ground should not much exceed 60° as they become very inefficient at higher angles.

In the final design, that is, when the precise stay tensions and pre-tensions are determined, it may be necessary to consider two cases with the wind in two different directions. However, the worst case occurs when the wind is parallel to one of the planes of the stays and blowing mid-way between the other two planes. The problem only occurs with three guy masts, four guy masts being equally resistant in all directions.

In three guy masts the wind load on any stay must be resolved parallel and normal to the plane of the stay and the former component only combined with the weight of the cable (stay) and the combination added vectorially to the normal component to find the effective value of w, the total load on the stay cable (Appendix 7).

Example.

Three stay masts 390 ft. high. Stays mutually inclined at 120° on plan and fixed to the mast at attachment points at 130 ft,. 260 ft. top levels. Deflection of mast at top not to exceed 20 in. either way.

The load is one of wind only, and on the top section of the mast, a wind load of 90 lb. per sq. ft. is taken. This is an ultimate load, and the mast (as usual practice for guyed masts) is to be designed for ultimate loads.

In order to find the wind loads on the masts and stays a preliminary design is necessary, based upon past experience with such structures, a suitable wind load being assumed, and by simple

resolution of the stay loads down and normal to the mast, it is a simple matter to find the compression in the mast done to the assumed wind loads.

A preliminary design of members can then be made, and in the present example, a triangular cross-section is chosen of 3 ft. 6 in. side with 3 in. leg members, and $1\frac{1}{2}$ in. bracing members, the design conditions being those prevailing in the heavily loaded base section.

The projected area for windage assuming bracing at 45° is just under 0·75 sq. ft. per foot of height for one face, and 1 in. dia. stays are indicated from the preliminary loading.

Wind load on stays (at 70 lb./sq. ft. basic load)
$= 70 \times 0{\cdot}085 \times 0{\cdot}7 \times 400 \times 3 = 4850$ lb.
Reaction at top of mast $= 2425$ lb.
The preliminary design is shown in Fig. 12.

The total reaction at each stay attachment point is the sum of the stay wind reaction and the point reaction due to the distribution of wind load on the mast itself. The latter may be obtained by considering the mast as a continuous beam supported at the stay attachment points.

Considering the top 130 ft. first, the ultimate wind load is :— $90 \times 130 \times 0{\cdot}75 = 8{,}775$ lb. on one face as the mast is triangular, multiply this single face load by 1·4 to give the total load. This is, therefore, 12,300 lb.

The wind loads on the mid and bottom sections are calculated in the same way as 9,600 lb. and 7,520 lb. respectively.

The mast is pin jointed at A and continuous at B and C and working from the loads on each span, the moments and reactions at the stay attachment points may be found by moment distribution (see Appendix 8). These reactions, plus one half the average wind loads on the stays themselves at each point, give the total horizontal load to be sustained by the stays at each point.

Station	A	B	C	D	
Load	7,600	9,600	12,300		lb.
Bending Moment	0	104,000	160,000	0	lb/ft.
Reaction	3,040	7,950	13,560	4,950	lb.
Wind Reaction At Tops of Stays	—	686	1,486	2,424	lb.
Horizontal Stay Load (total)	—	8,636	15,046	7,374	lb.

The table now gives the conditions calculated as above. It must be remembered that the method of distributing the moments

in the mast assumes that the mast remains straight when deflected. If, due to badly adjusted stays this is not so in practice, then the moments are likely to be much higher and the specified safety factor for mast design should take account of this.

Calculation of Full Tensions in Top Stays.

Weight of stay per foot = 1·86 lb.

Wind load per foot = 4·20 lb. (at 70 lb./sq. ft.).

To determine w_1, w_2, w_3, the wind load is resolved into components normal to and in the plane of the stay. (See Appendix 7).

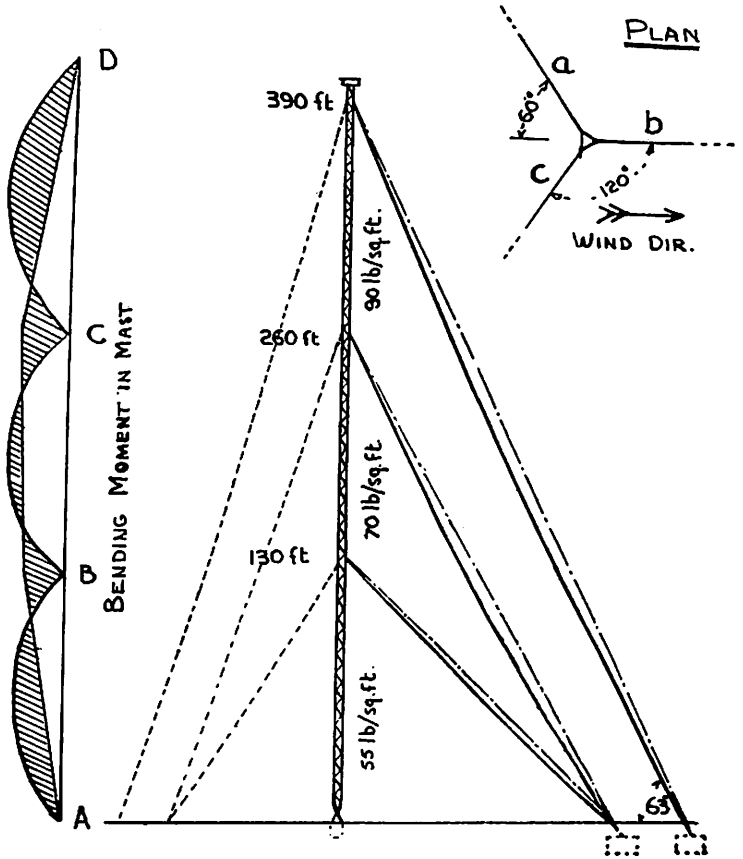


Fig. 12.

Referring to Fig. 12, the wind direction shown makes b the leeward stay, so :—

For stay b (leeward), $w_2 = 1{\cdot}86 \cos 63 - 4{\cdot}2 \sin 63 = -2{\cdot}90$ lb.

For stays a or c (windward), w_3

$$= \sqrt{(1{\cdot}86 \cos 63 + 4{\cdot}2 \sin 63 \cos 60)^2 + (4{\cdot}2 \sin 60)^2}$$

$$= 4{\cdot}5 \text{ lb.}$$

w (no wind) $= 1{\cdot}86 \times \cos 63 = 0{\cdot}85$ lb.

Elastic extension of stay per 1,000 lb. load = 0·04 ft. on the stay length of 438 ft.

The total sway at the top = 60 in. = 30 in. deflection.

$\therefore$ Allowable shortening of leeward stay $b = 30 \cos 63 = 13{\cdot}68$ in. (1·14 ft.).

Allowable lengthening of windward stay a, $c = 30 \cos 60 \cos 63$ in.

$= 0{\cdot}681$ ft.

Total $= 1{\cdot}14 + 0{\cdot}681 =$ **1·821 ft.**

Difference in stay tensions required $= \dfrac{7374}{\cos 63}$ (from table)

$= 16{,}200$ lb.

In this case it is to be noted that stays a and c, being at 120°, are considered as equivalent to one stay diametrically opposite b.

Choose $T_1 = 4{,}000$ lb. So $T_2 = 4{,}000 + 16{,}200 = 20{,}200$ lb.

Leeward stay $w_2 = -2{\cdot}9$ lb. $e = 4 \times 0{\cdot}04 = 0{\cdot}16$ ft.

$$D = \frac{w_2^2 l^3}{24 T_1^2} = 2{\cdot}45. \quad S_1 = e - D = \mathbf{-2{\cdot}29.}$$

Windward stay $w_3 = 4{\cdot}5$. $e = 20.2 \times 0{\cdot}04 = 0{\cdot}808$ ft.

$$D = \frac{w_3^2 l^3}{24 T_2^2} = 0{\cdot}174. \quad S_2 = e - D = \mathbf{0{\cdot}634.}$$

$$S_2 - S_1 = 0{\cdot}634 + 2{\cdot}29 = \mathbf{+2{\cdot}924.}$$

We require 1·821 ft. (= total change of stay length under load).

Difference is 1·103 (so recalculate leeward stay for D

$= 2{\cdot}45 - 1{\cdot}103 = 1{\cdot}347$).

So $T_1 = 5{,}400$ lb. T_2 becomes 21,600.

Leeward stay $E = 0{\cdot}216$. $D = 1{\cdot}347$. $e - D = S_1 = \mathbf{-1{\cdot}131.}$

Windward stay $E = 0{\cdot}864$. $D = 0{\cdot}152$. $e - D = S_2 = \mathbf{0{\cdot}712.}$

$$S_2 - S_1 = 0{\cdot}712 + 1{\cdot}131 = \mathbf{1.843 \text{ ft.}}$$

This is close enough to 1·821 and gives a deflection of 20¼ in.

The tensions are therefore 5,400 and 21,600 lb.

The total change in stay lengths (windward and leeward) is 1·843 instead of 1·821, so that under no wind conditions, that is when the mast is brought upright the leeward stay chord increases by $1{\cdot}14 \times \frac{1{\cdot}843}{1{\cdot}821} = 1{\cdot}154$ ft. Similarly stays *a* and *c* chords shorten by $0{\cdot}681 \times \frac{1{\cdot}843}{1{\cdot}821} = 0{\cdot}69$ ft.

$\therefore$ S (no wind) $= S_1 + 1{\cdot}154 = 1{\cdot}154 - 1{\cdot}131.$

or $S_2 - 0{\cdot}69 = 0{\cdot}712 - 0{\cdot}69.$

That is S = 0·023 ft.

T is the no wind or pre-tension in lb.

$$E = \frac{T}{1000}\, e \text{ and } D = \frac{w^{2}l^{3}}{24T^{2}}$$ where *w* is given the no wind value of 0·85 lb./ft.

$$E - D = S.$$

$$\therefore\ 0{\cdot}04T - \frac{0{\cdot}85^{2} \times 84}{24T^{2}} = 0{\cdot}023$$ (where T is in thousands of lbs.)

T = 4·2 or 4,200 lb.

In most cases T will lie between T_1 and T_2 unless the ultimate wind load is very high. It is also necessary to work out a similar calculation for the wind in the opposite direction, *b* becoming the leeward stay and *a* and *c* the windward stays. The resolved forces and loads are calculated from formulas 6 and 7 (Appendix 7) and again the tensions T_1, T_2 and T are found and the worst case is taken. The other stay sets are dealt with in the same way.

The Mast.

As a typical example, a mast for a three stay unit with the loading shown in Fig. 12*a* is taken. This loading is made up of the bending on the mast due to wind (bending load) and the vertical component of the stay tensions, to which has been added the estimated weight of the mast (vertical compression load).

Denote by S the distance on plan between the centroids of the main leg members.

Then $I_{xx} = I_{yy} = \frac{1}{2}\, AS^{2}$ where A = area of leg member.

Try S = 3 ft. 6 in. and A = 2·75 in.

$$I_{xx} = \frac{2{\cdot}75 \times 42^{2}}{2} = 2{,}425 \text{ in.}$$

$Z_j = \frac{I}{y}$. So $Z_{xx} = \frac{I_{xx}}{21} = 115$ in.^{3}

$$Z_{yy} = \frac{I_{yy}}{24{\cdot}25} = 100 \text{ in.}^3$$

r = radius of gyration $= \sqrt{\dfrac{2425}{2{\cdot}75 \times 4}} = 14{\cdot}85$ in.

$$\frac{l}{r} = \frac{122 \times 0{\cdot}85 \times 12}{14{\cdot}85} = 84.$$

By Johnson's formula.

Crippling stress $= f_p = 15\ (1 - 0{\cdot}000051 \times 84^2)$

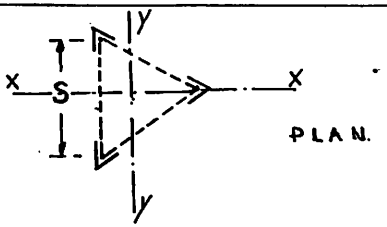
$= 8{\cdot}95$ tons/sq. in.

f_s Bending stress in mast $= \dfrac{66{,}000 \times 12}{100} = 7{,}920$ lbs./sq. in.

f_c Compressive ,, ,, ,, $= \dfrac{68{,}000}{3 \times 2{\cdot}75} = 8{,}250$ lbs./sq. in.

$$\frac{f_c}{F_c} + \frac{f_s}{F_s} = \frac{8250}{8{\cdot}95 \times 2240} + \frac{7920}{15 \times 2240} = \mathbf{0{\cdot}65.}$$ (This fraction must be < 1)

BNDG. MOMENT	STAY REACT. R	W. STAY LOAD	W MAST WT.	POSITION
				D
-153,000	16,670	32,500	2,500	C
-102,000 +66,000	5,000	6,000	2,500	B A
lb.-ft.	lb.	lb.	lb.	



PLAN.

Fig. 12 (a).

The max. compressive load in one leg = 16,170 lb. = 7·22 tons.

max. unsupported leg length = 4′ 0″ (from tables).

The maximum shear is 15,200 lb. and the whole of this load in a triangular mast is taken on one face.

So that load in horizontal brace = 15,200 lb.

Load in diagonal (tension member) = $\sqrt{2} \times 15{,}200$ lb.

= 21.500 lb,

It should be noticed for mast latticed sections, triangular or square, the moment of inertia is the same about both axes. The modulus of section, however, varies and when assessing stresses due to bending moments, the correct value must be taken.

Notes on Detailing (All Types of Towers).

The whole of one side face of the tower is treated as a plane frame, so that if the leg slopes are small the frame is detailed exactly as shown on the elevation by making a large scale detail or calculating the lengths of the members. In any case large details of the joints will be required to determine any end corrections due to rivet or bolt connections.

Changes in the leg angles affect only the joints in the main legs, and are taken care of by utilising special fish plates in which one set of holes is drilled on the skew.

With tubular main legs, flange plates are used, and in this case the top flange of a joint is set at the correct cant in the front face and the mating flange in the side face, so that multiple angles are avoided. The flanges are notched or otherwise marked for identification of the direction of slope.

It is necessary to make geometrical developments of such details as :—

(1) Cross arms triangular on plan.
(2) Hip bracing.
(3) Triangular top sections.

The developments will give the lengths of the members but joints should receive special attention either on the board or in the shop, as awkward angles are involved in many cases.

It is important that suitable packing pieces of the correct thickness are used at points where bracing members cross. This is to ensure that all struts are co-planar and are not pulled into position during erection, as any residual lack of straightness obviously invalidates design based upon crippling load formulae. The first diagram of Fig. 8 illustrates this, the remaining diagram being of a low-level leg joint where bracing members attach to a large angle leg section, and a typical welded joint for flange connected members as used in tubular construction. With this type of construction much larger members may be used for the same

economy of material, and the bracing members may, if the design permits, be much more widely spaced. With smaller tubular towers, an alternative joint consists of welding a gusset plate on the side of the tube leg, and insert plates into the ends of the bracing tubes thus giving a standard flat plate joint with bolts in single shear.

Foundations.

Tower foundation may be required to take vertical loads, both down and uplift together, or one or other of these separately, and in addition, side loads.

For example the mast foundation of a guyed mast takes vertical down loads only and no uplift whilst the stay anchorages take uplift and side force only.

The foundation blocks of fixed base self-supporting towers take both downward and, to a lesser extent, upward loads alternately.

In the latter case the foundation may be of concrete, or, in the case of transmission line towers up to about 150 ft. in height an extension leg and steel plate or grid is used if the ground is not dry and sandy in nature. In this case the weight of the soil above the plate balances the upthrust and it is permissible to base the design on a weight of earth in the shape of a frustum of a pyramid the sides of which make an angle of between 50° and 60° with the horizontal. In the case of a concrete block foundation, the upthrust is balanced by the weight of the block. The upthrust used for design of the foundation is usually twice the vertical reaction due to the normal minimum loading, which, of course, also applies to the downward loading. The figures given above, however, are generally specified by the client. (See also B.S.S. 449 of 1948, p. 32, Ref. No. 3).

In some cases a single concrete block is used for the four feet of the structure (raft foundation). This is impracticable in all cases except where a narrow base is used, and although it has been used on large towers where special conditions prevail, such as the possibility of earthquake shocks, the use of a raft is very uneconomical in such cases. It may be used on very small bases to advantage as it prevents the excessive stresses set up in the structure as a result of uneven settlement of the feet where separate blocks are used.

Example.

Separate foundations.

Tensile load (normal)	=	26 tons
Compressive load (normal)	=	50 tons
Safe bearing on ground	=	2 ton/sq. ft.

(1) Concrete block.

Base area = 50/2 = 25·0 sq. ft.

$$\text{Volume of concrete} = \frac{2 \times 26 \times 2240}{150} = 77{\cdot}7 \text{ c.ft.}$$

$$\text{Depth} = \frac{77{\cdot}7}{25{\cdot}0} = \mathbf{3{\cdot}15\ ft.}$$

(2) Plate. (See Figs. 14 and 15).

Base area = 25 sq. ft. as above.

Let d = depth of plate below ground.

Then weight of earth above (60° angle).

$$W = \frac{d\,(5+d/2)^2 \times 90}{2240} = d + 0{\cdot}201d^2 + 0{\cdot}01d^3 \text{ tons.}$$

d is therefore given by

$$2 \times 26 = 0{\cdot}201d^2 + 0{\cdot}01d^3 + d.$$

$$d = 11{\cdot}5 \text{ ft.}$$

Bases Subject to Horizontal and Vertical Loads.

An example in the stay anchorage block of a guyed mast (Fig. 13). The inclined upward load due to stay tension when resolved into its vertical and horizontal components gives rise to a wholly vertical force balanced by the weight of the block and a heavy side load

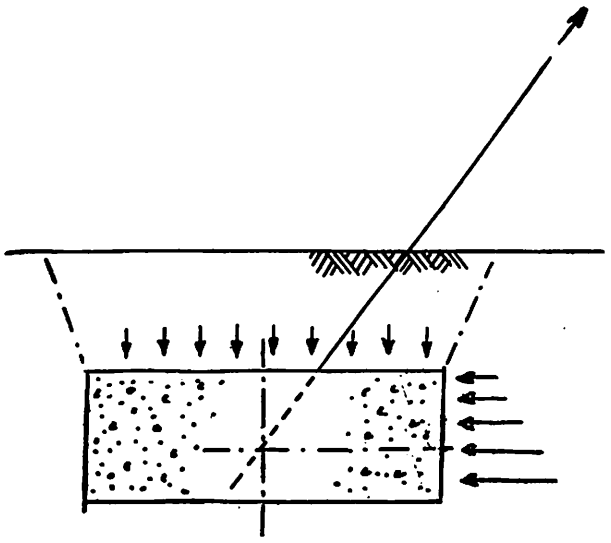


Fig. 13.

balanced by direct pressure of the ground on the end of the block and friction forces at bottom and sides. The latter can generally be ignored, as, owing to the preponderance of the vertical force, weight considerations alone will necessitate the size of the block being large enough to enable all horizontal loads to be taken on the end face.

The block should be so designed that the line of action of the horizontal resistance, which may be below the centre line due to ground resistance increasing with depth, and the line of the stay tension meet at a point immediately below the centre of gravity of the block.

The deeper the block can be set, the greater will be its resistance both to horizontal and vertical movement so that a smaller block may be used at greater depths.

Raft Foundations.

In the simplest case (Fig. 13*a*) the earth pressure under the base may be considered to be made up of two parts. An even pressure equal to w/d^2 where w = weight of block plus tower and vertical loads, and a pressure varying from $-f$ at one side to $+f$ at the other side due to the overturning moment M.

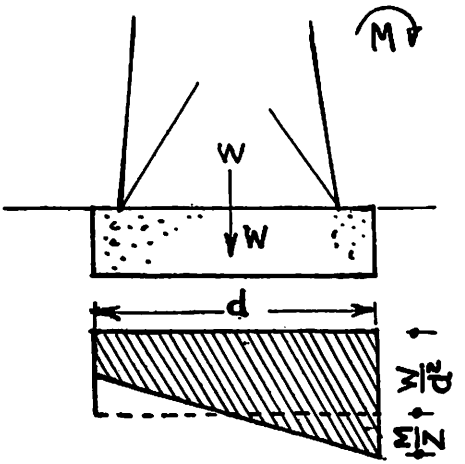


Fig. 13 (a).—Conservative design distribution of load for maximum overturning.

For any base f as defined $= \frac{M}{Z}$

and for a square base $Z = \frac{d^3}{6}$

For optimum conditions, that is where the tensile stress due to M is just balanced by the direct pressure due to W.

$$\text{Then } \frac{W}{d^2} = \frac{M}{Z} = \frac{6M}{d^3}$$

$$\text{or } W = \frac{6M}{d}$$

W can, of course, exceed this value but must not be less if the whole of the base of the block bears on the earth.

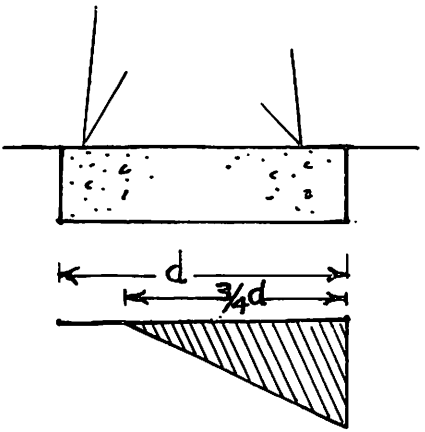


Fig. 13 (b).—Worst allowable condition of load distribution. Practical designs should lie between these two cases.

To find the bearing pressure at the worst point on the base add f to $\frac{W}{d^2}$.

Example.

$W = 60$ tons. $M = 170$ tons ft.

$$W > \frac{6M}{d}$$

$$60 > \frac{1020}{d}$$

$$\text{or } d > \frac{1020}{60} = 17 \text{ ft.}$$

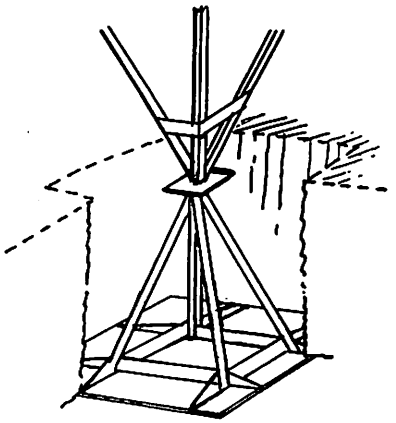


Fig. 14.

Taking the minimum size, the maximum bearing pressure is given by adding $\frac{170}{(17^3/6)}$ and $\frac{60}{17^2}$

$= 0{\cdot}42$ tons /sq. ft.

Another method is illustrated in Fig. 13*b*, in which it is assumed that no tension or compression exists in the left hand part of the base.

It is necessary, however, to find the optimum value of the fraction n to give the maximum resistance to an overturning moment for a given value of maximum ground bearing load.

Put this load equal to unity.

The upward thrust per ft. of width $= \frac{nd}{2}$

and the lever arm $= \frac{d}{2} - \frac{nd}{3}$

So M (per ft. of width) $= \frac{nd}{2}\left(\frac{d}{2} - \frac{nd}{3}\right)$

$$M = \frac{nd^2}{4} - \frac{n^2d^2}{6}$$

$$\frac{dM}{dn} = \frac{d^2}{4} - \frac{2nd^2}{6}$$

for M to be a maximum $\frac{dM}{dn} = 0$

$$\frac{d^2}{4} = \frac{2nd^2}{6}$$

$$n = \frac{3}{4}$$

So that if f is the maximum bearing pressure on soil, the upward load is $\frac{3}{8}fd^2$ tons, and the lever arm of this soil reaction is $d/4$.

Taking the previous example

W = 60 tons. M = 170 tons.

$$\tfrac{1}{2}f \times \tfrac{3}{4} \times d \times d = 60 \quad (1)$$

$$60 = d/4 = 170 \quad (2)$$

from which

$d = 11{\cdot}3$ ft.

$f = 1{\cdot}25$ tons/sq. ft.

If, in addition, a side thrust exists on the foundation, as is normally the case, due to torsion and other side loads, the side bearing pressure must also be considered. Normally, it is possible to take the side bearing pressure on the soil as about 25% of the vertical bearing load intensity *f*. The determination of bearing loads in practical cases, falls within the realm of the new science of soil mechanics, and for further information the reader may consult one of the standard practical treatises on the subject.

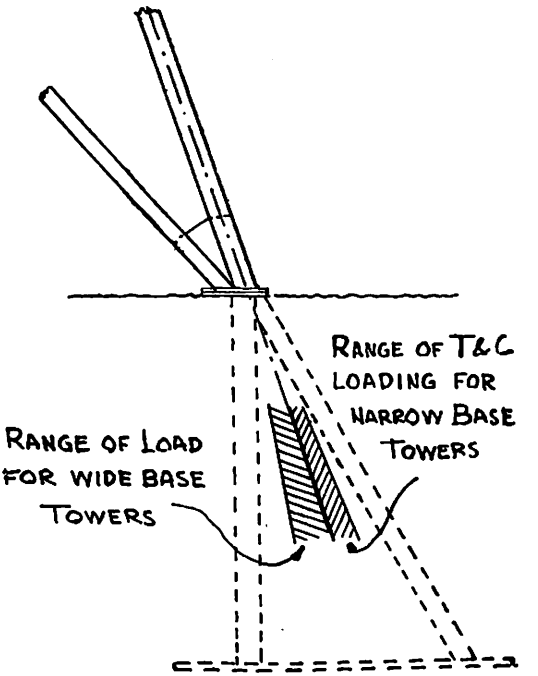


Fig. 15.

APPENDIX 1.

Torsion on Lattice Towers.

A tapered tower of height y to apex and base width ay has a force P applied at level $(y-z)$ and at a distance x from the centre (Fig. 16).

The load P may be replaced by a horizontal load on each face of $\frac{Px}{2\,(az)}$, that is, a total torsion of $4 \times \frac{Px}{2\,(az)} \times \frac{(az)}{2}$ since the lever arm is $\frac{(az)}{2}$. This expression is equal to Px and so is equivalent to the torsion. In order completely to balance the load P two further loads are to be applied to the side faces equal to $\frac{P}{2}$ each.

In the same way by considering the torsion reaction at the base of the tower, the equivalent torsion load on each face is $\frac{Px}{2\,(ay)}$ since the base width is (ay) and the lever arm increased to $\frac{(ay)}{2}$

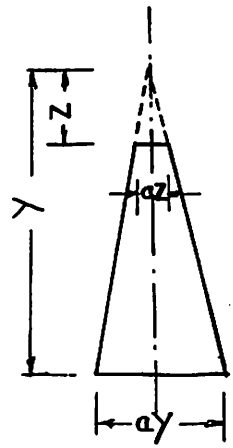


Fig. 16.

Returning to the first expression and analysing the face by the method of sections :—

Taking moments about apex.

$$\text{Shear S at base} = \frac{Px}{2(az)} \times z \times \frac{1}{y}$$

$$= \frac{Px}{2(ay)}$$

which is equal to the base torsion load as calculated from the second expression.

Direct analysis of a face with the equivalent torsion and direct loads as defined above therefore gives the correct bracing loads due to torsion, and the analysis may be carried out by calculation or stress diagram whether the tower is tapered or not.

APPENDIX 2.

Graphical Design.

Fig. 17. Single Circuit Suspension Type.

Vertical and side (wind) loads only. The loading condition may be defined as follows :—

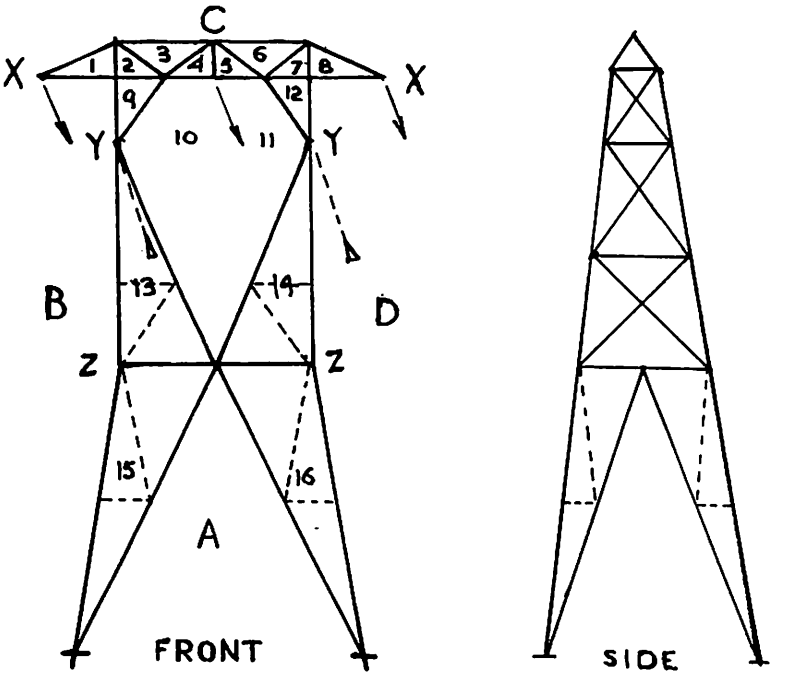
(1) Weight of wires with ice loading and wind load on wires and structure.

(2) Broken wire condition centre wire.

(3) Broken wire condition outer wire.

In this type of structure the graphical analysis of case (1) can only be undertaken by first determining the reactions at YY. This is done by proportioning the vertical loads and taking moments about Y for the horizontal loads. The load diagram may then be drawn as shown on the left for the top section and a separate diagram on the right for the base section. The first one is self-closing and therefore self-checking ; the second must be checked by calculation of the base reactions by taking moments about one of the bases. Because the members in the top frame XX are sloping considerably the loads as scaled in this section must be multiplied by cos θ where θ is the angle of slope with the vertical.

Case (2) loads the horizontal plane frame XX which loads may be easily determined by calculation, and half the broken wire load is transferred to each side face and a stress diagram for the side face for unit load at the top, may be multipled by the value of the actual load to determine the member loads.


C
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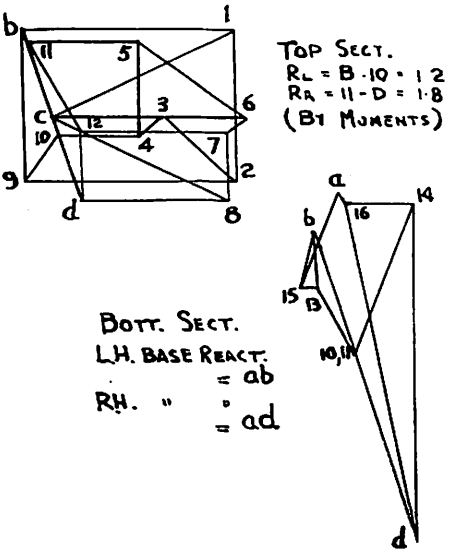
13
14
15
16
FRONT
SIDE


b
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d
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TOP SECT.
RL = B·10 = 1·2
RR = 11-D = 1·8
(BY MOMENTS)
a
14
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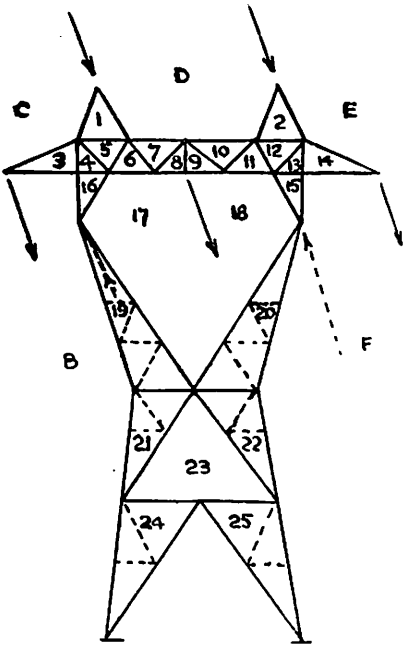
10,11
d
BOTT. SECT.
LH. BASE REACT. = ab
RH. " " = ad

Fig. 17.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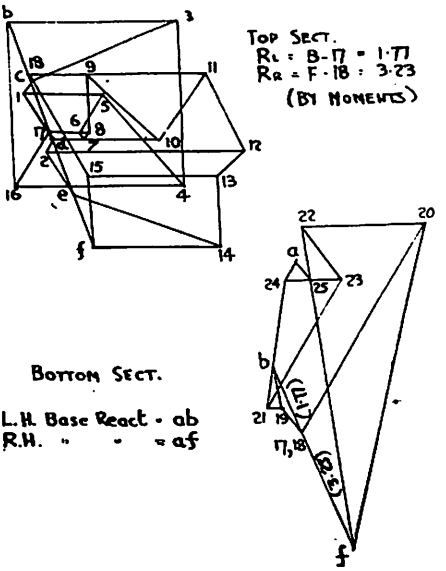

Top Sect.
R_L = B-17 = 1·77
R_R = F-18 = 3·23
(By Moments)
Bottom Sect.
L.H. Base React = ab
R.H. " " = af
(11·1)
(12·3)

Fig. 18.

Case (3) involves a fourth analysis of the horizontal frame XX, the loading being different from case (2), and a much larger reaction is now taken down the side face (owing to the narrow rectangular shape on plan, the whole torsion may be considered as coming on the side faces) and the unit diagram used in case (2) may be used again, with the much larger factor, to determine the member loads in this case.

Fig. 18. High Voltage Single Circuit Type.

Four cases to be considered, the fourth condition being a broken earth wire.

The stress diagram for the dead + wind loading shown for the front face is given.

Brace Loads.

In this type of tower, some of the members may achieve their maximum loading in tension, and in tabulating both tension and compression loads must be recorded. Any member on the left of the tower face must also be paired with the corresponding members on the right and the worst case taken, *i.e.*, C – 3 to E-14, and 16-17 to 15-18.

The front face and side face loads are not, of course, interconnected.

Leg Loads.

In this case side and front faces are additive, that is the worst compressive load in, say, F-22 (front) is added to the worst compressive load in the same member from the side face diagrams.

APPENDIX 3.

Graphical Design of Double Circuit Suspension Tower.

Figs. 19 and 21 represent the basic line diagram for the loading systems, and the "stress" diagrams for two double circuit transmission towers. Fig. 19, the full design diagrams for a simple type of suspension pylon for 110 Kv. 0·6 aluminium sheathed steel cable.

Typical dimensions for a normal type would be as follows :— 50 ft. to bottom cross-arm, 62 ft. and 73 ft. to the top ones, and 80 ft. to the extended top at which point the earth wire is carried. The type given is a short base tower which is often used for crossing hill spurs, and owing to its simplicity, the diagram may be easily followed. Most modern transmission lines are for 132, 220, 275 384 Kv. or even higher voltages, and are correspondingly more

complicated structurally. All loads are related to unit vertical cross-arm load, and the other loads factored to correspond with this. For instance, the vertical load for 800 ft. spans with ice loading would be of the order of 10 cwt., or correspondingly less in areas where no ice loading need be considered. The specified wind loading also affects this figure. There is consequently some point in drawing up for unit loadings, the necessary correction being applied during tabulation of loads.

Fig. 21 shows a larger type for 220 Kv. of overall height of 90 to 100 ft.

The first stage in graphical design is the numbering up for the load or stress diagram. Secondary bracing or support bracing (chain dotted) is ignored, and redundant members (dotted) cannot be analysed by graphic statics ; they will, however, reduce the load carried by the calculable members, and their correct proportion of the load determined at the tabulation stage, a usual method being merely to divide the load equally.

Loading Conditions.

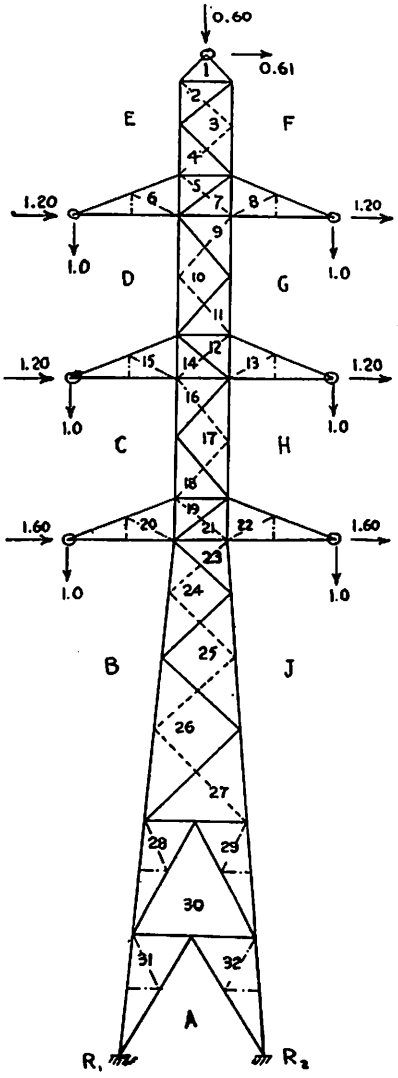
(1) Fig. 19 (*a*). Weight of lines, plus wind load on lines and structure, the latter being added to the bottom cross-arm load.

(2) Fig. 19 (*b*). Broken line condition plus vertical loads, but loads on top cross-arm reduced. The equivalent face load is determined for load T (parallel to line) and inserted as shown.

(3) Fig. 19 (*c*). Broken earth wire at top of tower.

Fig. 19 (*d*) shows the stress diagram for case (1). In this case the diagram may be completely drawn without determining any reactions, but it is not self-closing so that in order to check the accuracy of the work, it is necessary to calculate the base reactions. The side thrusts on the bases are not equal so that it is necessary to apply the method of sections to the base panel and legs in turn. The brace loads are equal and opposite. The leg loads are found by taking moments about the brace meeting point, and one half of the vertical load is added to each leg in order to correct for vertical loading. The brace and leg loads are added vectorially to obtain the base reactions. These must be equal in magnitude and direction to the loads scaled by *a-b* and *a-j*.

Case (2) involves finding the worst cross-arm at which a broken wire can occur. This may be different for different parts of the tower. The rule is quite simple. At any section, produce the legs to meet at a point. The cross-arm farthest from that point is the one giving greatest brace loads at that section when the line breaks at that cross-arm. Fig. 19 (*e*) is part of the stress diagram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R1
R2

Fig. 19 (a).

for broken line at top plus normal loadings, however reduced vertical and side loads are specified (as a general rule) at the point of application of the torsion load, and the torsion equivalent load on each face is inserted in the diagram as g_1-g, representing a horizontal load applied at the face at the bottom level of the top cross-arm. Fig. 19 (*e*).

The third diagram is best drawn for unit load at the top and can, by using the appropriate factor be used for analysing the side face for case (2) and case (3). Fig. 19 (*f*).

Brace loads are taken from *d* or *e* for the front face and for the side face *e* and *f*, the greater values being taken. Leg loads are taken from *d* and *f* and the values from the two diagrams must be *added*.

Notes on Graphical Analysis.

In order to ensure that a stress diagram of any plane frame is completed without errors or difficulties, it is important to carry out the work in stages and each stage completed before commencing the next. To this end the following notes may be found helpful :—

(1) Since neither redundant nor deficient frames can be analysed by graphics, the frame should be examined to ensure that all redundant members are removed (*i.e.*, that the form of bracing is of Z, K or N formation) and that the frame is completely formed of simple triangles and is therefore not deficient.

(2) Number and letter the frame (numbers may be used in each triangle and letters between loads) so that any single member has a number or letter on either side and each load and reaction has a letter on either side.

It is evident that the structure as a whole must be in equilibrium and the loads and reactions must themselves form a closed diagram. If this were not so, it would not be possible to "close" the final diagram accurately. One source of error may be eliminated by drafting the equilibrium diagram first and constructing the final diagram around it. Care should also be taken in working in one direction only around each structural joint in turn and numbering the points on the diagram as the work proceeds.

It will be found in some cases (*e.g.*, the designs of Figs. 19 and 21 and the bottom sections of Figs. 17 and 18) that the diagram may be completed without first finding the base reactions as these come out in the diagram automatically. If there is any doubt, however, (and they will have to be calculated as a check) they should be inserted at the beginning.

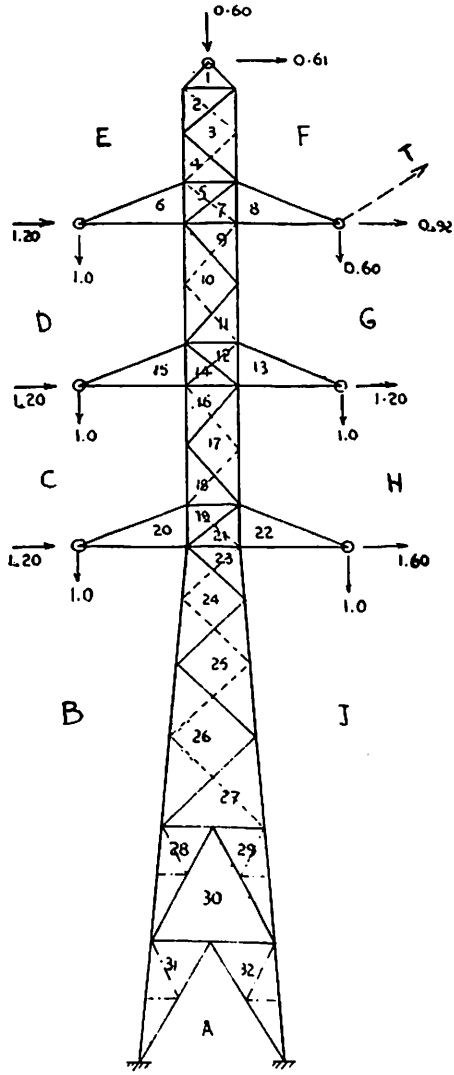


Fig. 19 (b).

APPENDIX 4.

Summary of Formulas.

Determination of the buckling load of a strut.

(*a*) Very long struts $L/r > 250$.

$$\text{Euler formula}: \quad P = \frac{\pi^2 EI}{L^2} = Af_e$$

(*b*) Moderately long struts L/r between 100 and 250.

$$\text{Johnson formula}: \quad P = f_y A \left\{ 1 - g \left(\frac{L}{r} \right)^2 \right\}$$

(*c*) Perry-Robertson formula :

$$\frac{P}{A} = \frac{f_y + (\eta + 1) f_e}{2} - \sqrt{\left\{ \frac{f_y + (\eta + 1) f_e}{2} \right\}^2 - f_y f_e}$$

E = Young's Modulus in tons/sq. in.

I = Moment of Inertia of strut section (minimum) in ins.^{4}.

A = Cross Section area of strut in ins.^{2}.

L = Effective length of strut in ins. = lQ.

P = Ultimate strut load in tons (equivalent to buckling load).

r = $\sqrt{\frac{I}{A}}$ = minimum radius of gyration of strut section.

f_e = Eulerian value of buckling stress tons/sq. in.

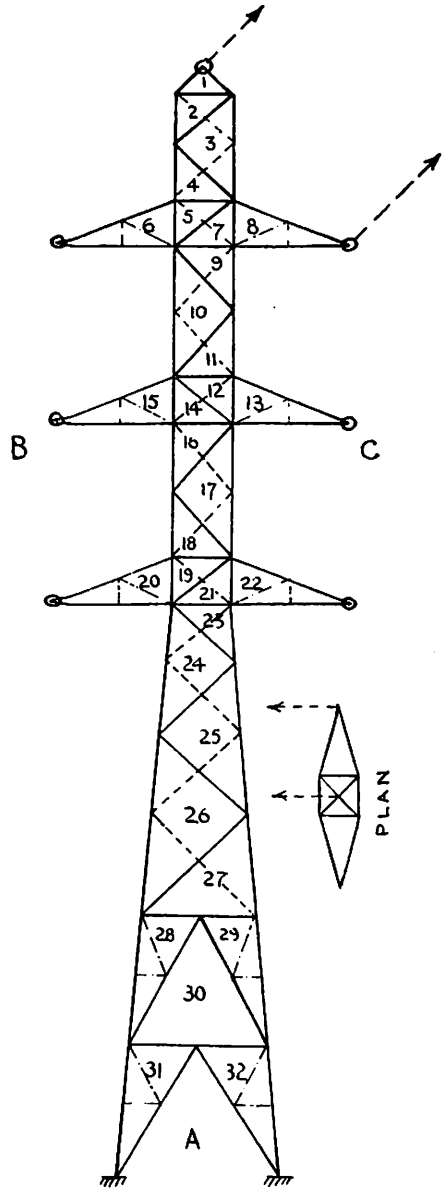
f_y = Minimum yield stress of material tons/sq. in.

η = $0{\cdot}003 \times L/r$.

g = A constant (0·000057 for mild steel).

l = Actual measured length of strut, ins.

Q = Factor varying from 0·7 for fixed ends to 1·0 for pin ended strut.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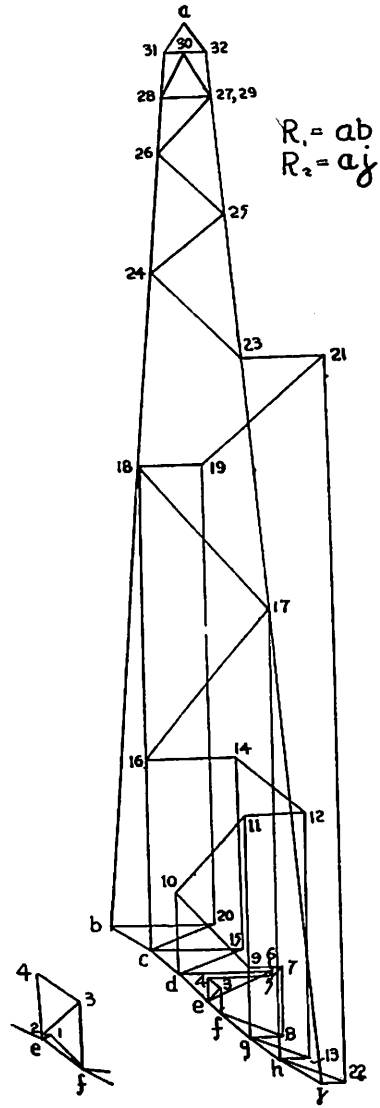
PLAN

Fig. 19 (c).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c
d
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22
R₁ = ab
R₂ = aj
4
3
2
1
e
f

Fig. 19 (d).

APPENDIX 5.

Wind loads on towers.

$$P = 0{\cdot}0042\ V^2$$

Where P = wind load in lb. per sq. ft. of flat projected surface.

V = velocity of wind m.p.h.

Maximum Ultimate Wind Load Table.

Height Above Ground, ft.	Wind V m.p.h.	Press lb./sq. ft.
0	109	50
100	119	60
200	129	70
300	138	80
400	146	90
500	154	100
600	162	110
700	169	120
800	176	130
900	182	140
1000	189	150

When designing on safe loads and not ultimate loads, one half the above figures may be taken.

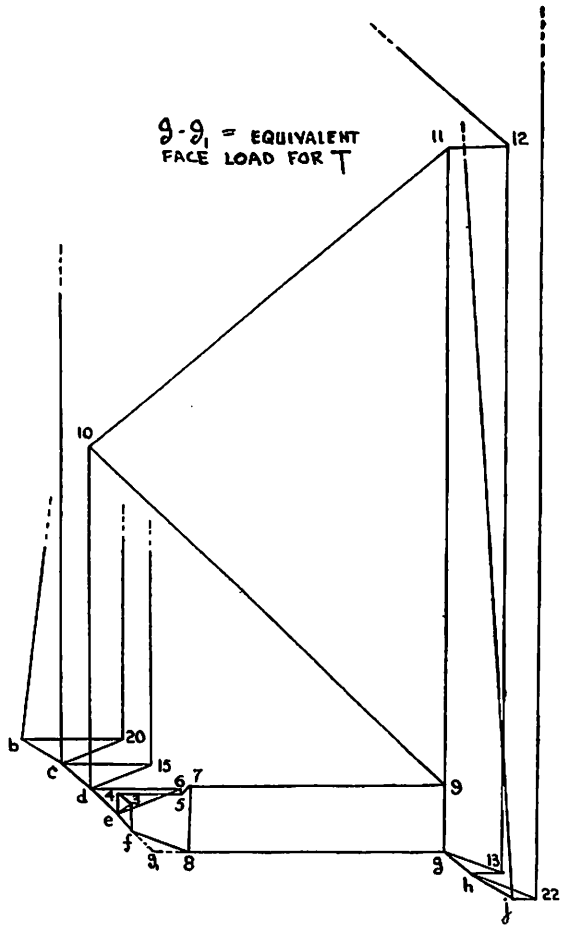
g-g1 = EQUIVALENT FACE LOAD FOR T
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Fig. 19 (e).

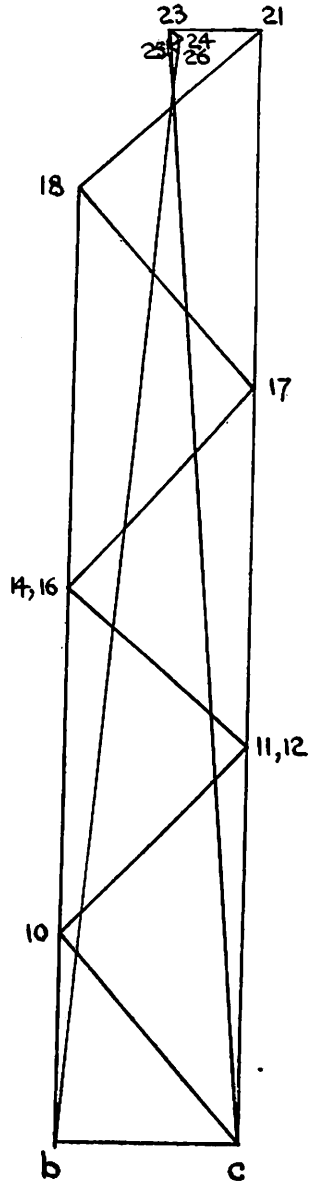

Fig. 19 (f).

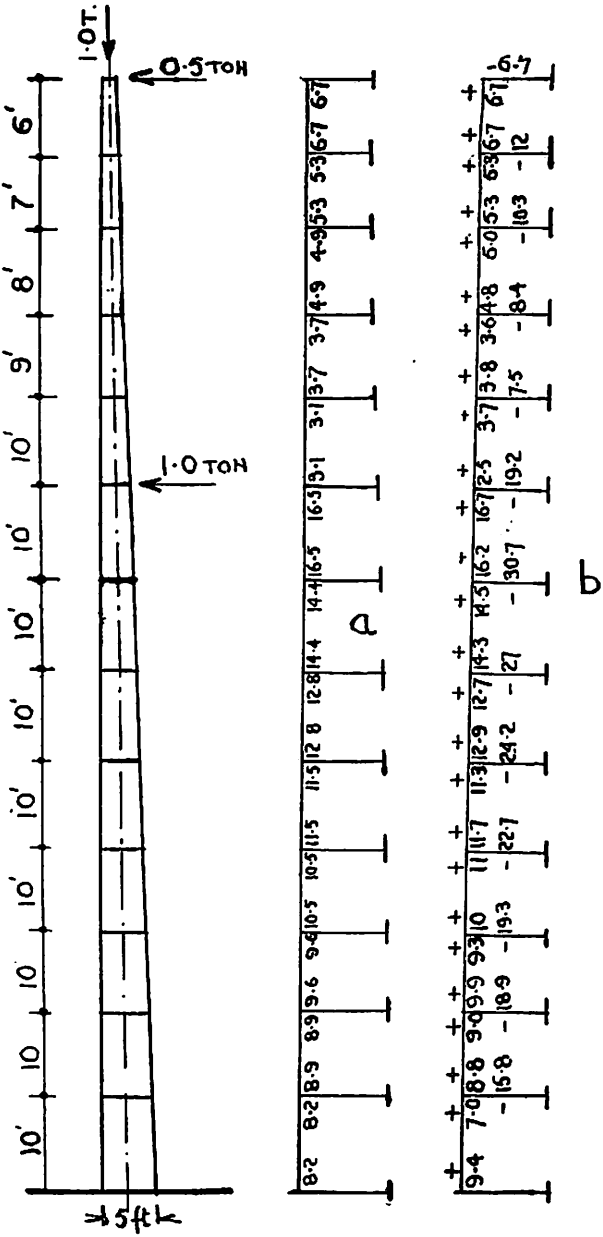

Fig. 20.

APPENDIX 6.

Design of Vierendeel or Open Frame Mast. (Fig. 20).

This type of design is often more economical for comparatively small structures and light loading. There is no diagonal bracing, the shear load being taken on the main legs. There are considerable bending moments at the joints and this design is therefore mainly used for welded masts.

The mast shown in Fig. 20 is 120 ft. high, 5 ft. at the base and 1 ft. at the top, square on plan.

The bending moments at the joints will be calculated :—

Meeting point of legs = 120 + 120/5 = 144 ft. above ground. Taking moments about this point and denoting by S the residual shear, that is, the shear force taken by the bracing, or total shear force less the total horizontal components of the tension and compression in the main leg members :—

$$\text{Top Section :—} \tfrac{1}{2} \times 24 = S\,(144 - x) \text{ or } S = \frac{12}{144 - x}$$

$$\text{Bottom Section :—} \tfrac{1}{2} \times 24 + 1 \times 64 = S\,(144 - x)$$

$$\text{or } S = \frac{76}{144 - x}$$

There is an initial bending moment in each leg panel varying from M at the top to plus M at the bottom.

$$\text{Where } M = \frac{S}{2} \times \text{panel height, or for each leg}$$

$$\frac{S}{8} \times \text{panel height for a square mast.}$$

$$\text{Thus for bottom 10 ft. panel } x = 5 \text{ and } S = \frac{76}{139} = 0{\cdot}547 \text{ tons}$$

$$\text{and } M = \frac{5{\cdot}47}{8} \text{ tons ft.} = \frac{5{\cdot}47 \times 12}{8} = 8{\cdot}2 \text{ tons in.}$$

$$\text{2nd panel } x = 15. \quad S = \frac{76}{129}. \quad M = 8{\cdot}9 \text{ tons in.}$$

All these initial moments are plotted for one side of the face in Fig. 20 (*a*). They are all considered positive since by inspection of Fig. 20 (*c*), which shows the form of deflection, the leg moments at the joint are both anti-clockwise and are to be balanced by a moment (clockwise or negative) in the cross member.

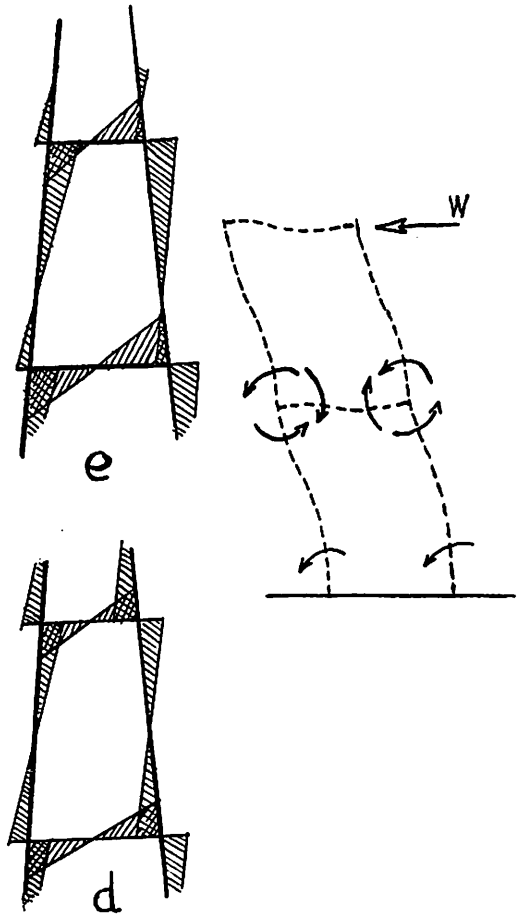

Fig. 20 (c)

These moments may be distributed by moment distribution. (See Reference 6).

Assuming that the cross members have the same I value as the main leg members, the final distribution is shown in Fig. 20 (*b*) for the left half of the mast face. The right half is exactly similar.

As a check on the moment distribution work, the sum of the moments at each joint must be zero, and the sum of the moments at the top and bottom of each panel must remain unchanged, although the moments themselves change. The reason for the first of these conditions is that the joint must be in equilibrium, and for the second that the sum of the moments in a leg panel is the total change in bending moment and this divided by the panel height is $\frac{dM}{dx}$ or the shear in the leg which in turn is determined solely by the loads on the mast. The B.M. diagram for a typical case is sketched in Fig. 20 (*e*) for two panels.

If, instead of the cross members being of the same section as the legs, they are considered smaller, the final moments in the structure will be differently distributed, a greater variation appearing in the positive moments in the legs and the negative brace moments being reduced.

The mast of Fig. 20 is not the most economical, as, by inspecting the moments calculated on the assumption of equal proportions, the cross members are over stressed.

On recalculating for larger cross members the B.M. diagram will be more like Fig. 20 (*d*), a "shift" in the diagram for the main legs taking place and decreasing the maximum B.M. on the leg.

If absolute structural efficiency is aimed at, the ultimate design can only be arrived at by trial and error, but much work should not be necessary in most cases.

APPENDIX 7.

Guyed Mast with three stays at 120° on plan. (See Fig. 12).

θ = Angle of stay with ground.
δ = Deflection at top of mast ft.
P = Average value of wind pressure on stay lb./sq. ft.
a = Projected area of stay cable sq. ft./ft.
W = Dead weight of stay cable lb./ft.
w = Component of W normal to stay.
w_2 = Total resolved load per ft. normal to leeward stay lb.

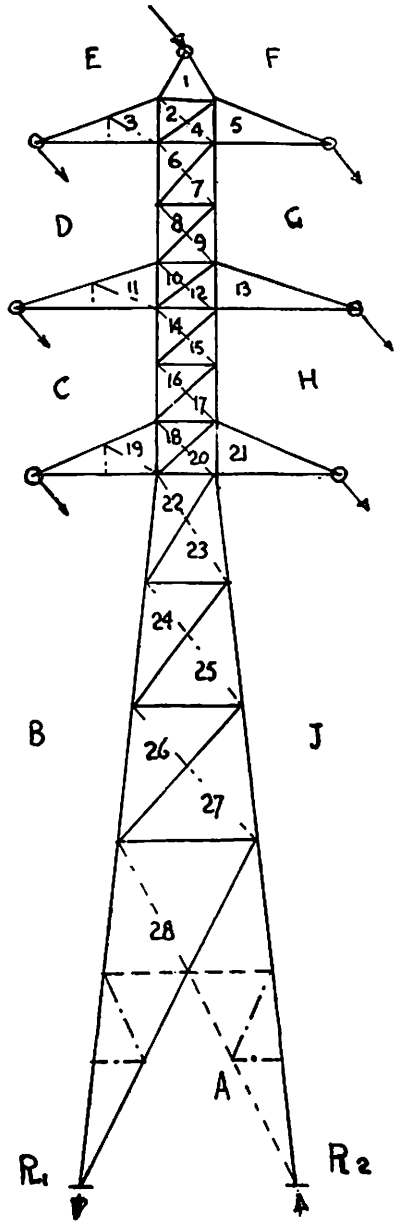
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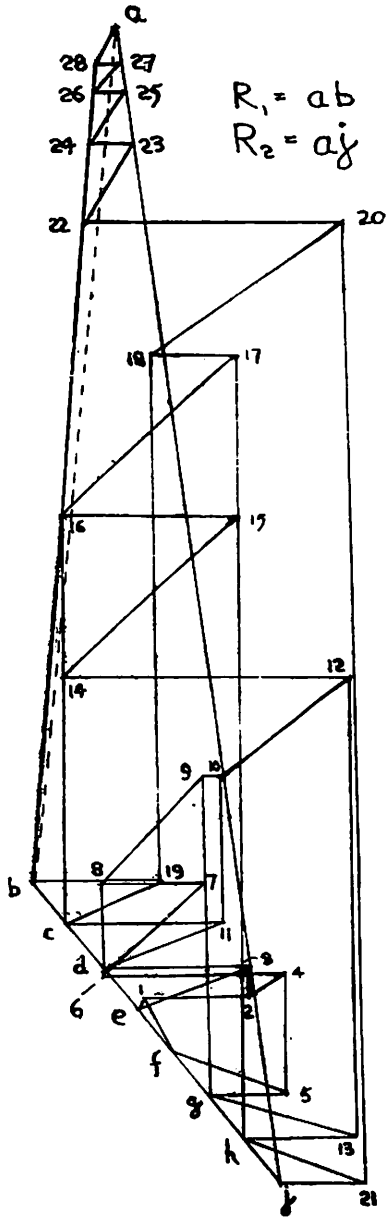
Fig. 21.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$R_1 = ab$
$R_2 = aj$

Fig. 21 (a).

w_1 = Total resolved load per ft normal to windward stay lb.

d = Alteration in length of stay corresponding to δ ft.

(1) d (stay b) $= \delta \cos \theta$.

(2) d (stay a or c) $= \dfrac{\delta \cos \theta}{2}$

(3) $w = W \cos \theta$.

(4) w_2 (stay b) $= w - aP \sin \theta$.

(5) w_1 (stays a or c)

$$= \sqrt{(w + aP \sin \theta \cos 60°)^2 + (aP \cos 30°)^2}$$

with wind in opposite direction.

(6) w_1 (stay b) $= w + aP \sin \theta$.

(7) w_2 (stays a or c)

$$= \sqrt{(w - aP \sin \theta \cos 60°)^2 + (aP \cos 30°)^2}$$

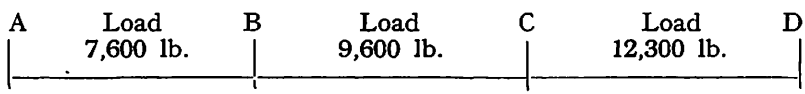
(8) T_1 = leeward stay tension.

(9) T_2 = windward stay tension.

(10) T = stay pre-tension.

APPENDIX 8.

(Moment Distribution in Long Narrow Mast).



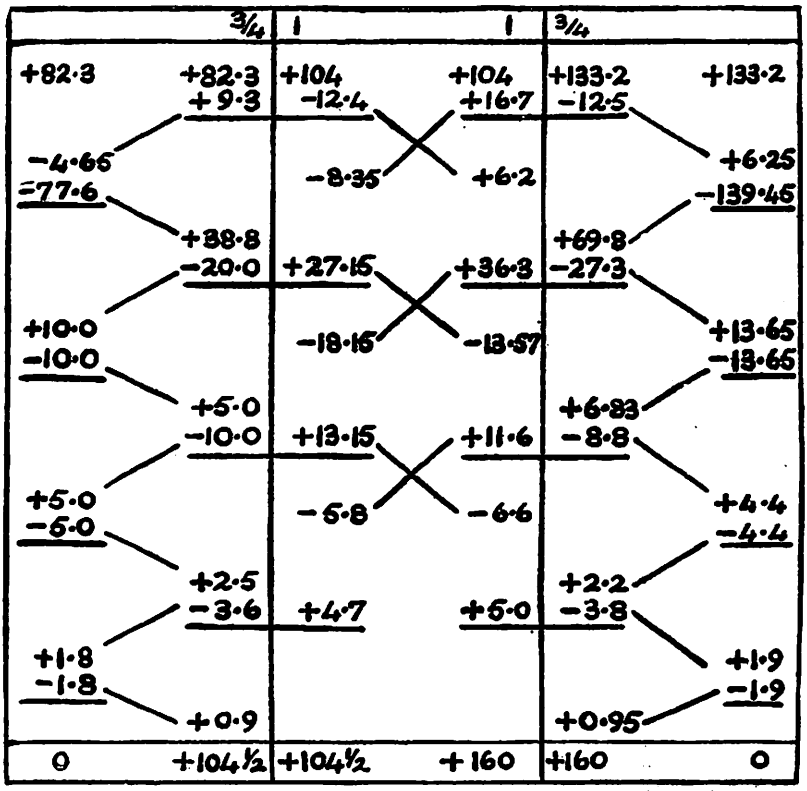
F.E.M.s $\dfrac{7600 \times 130}{12} = 82{,}300$ lb. ft.

$\dfrac{9600 \times 130}{12} = 104{,}000$ lb. ft.

$\dfrac{12300 \times 130}{12} = 133{,}200$ lb. ft.

Distribution factors—centre span = 1.

end spans = $\frac{3}{4}$.



Having found the moments, the reactions are easily determined by taking moments about B and C working from A and D in turn.

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25. $\frac{7}{8}$" ,, ,, ,, (30 ton yield).
26. 1" ,, ,, ,, (30 ton yield).
27. $\frac{3}{4}$" ,, ,, ,, (40 ton yield).
28. $\frac{7}{8}$" ,, ,, ,, (40 ton yield).
29. 1" ,, ,, ,, (40 ton yield).
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